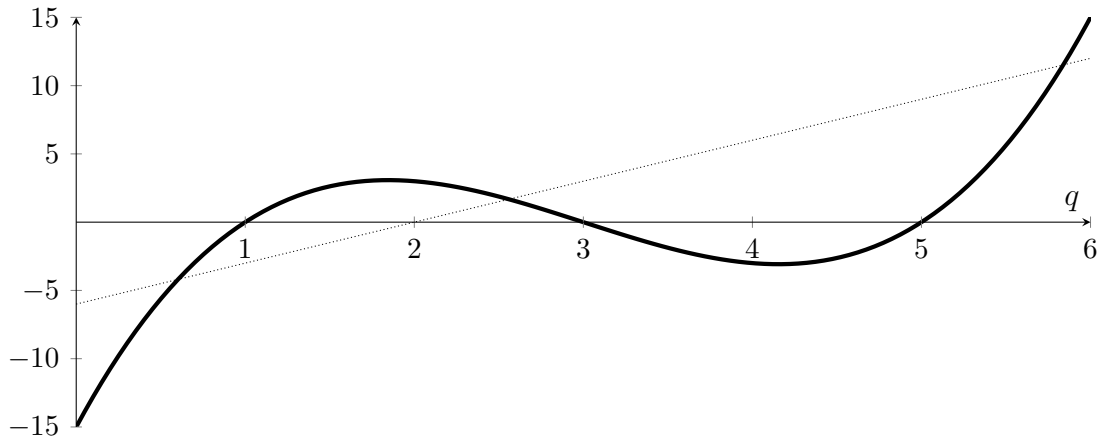


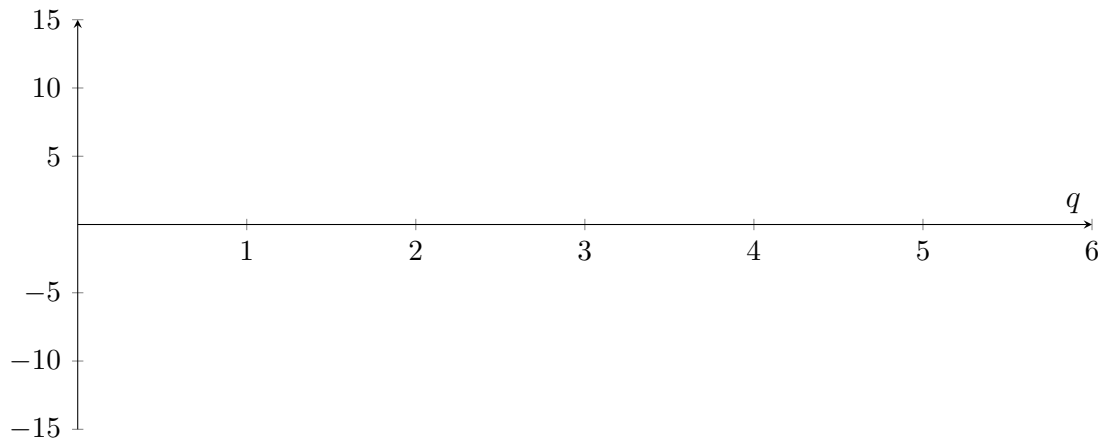
1. The graphs of the two functions $y = 3x - 6$ and $y = x^3 - 9x^2 + 23x - 15$ are below.



The piecewise-defined function $P(x)$ is given by:

$$P(x) = \begin{cases} 3x - 6 & \text{if } 0 \leq x < 3 \\ x^3 - 9x^2 + 23x - 15 & \text{if } 3 \leq x \leq 6 \end{cases}$$

- (a) (10 points) Sketch the graph of $y = P(x)$.



- (b) (4 points) The range of $P(x)$ is _____ (use interval notation)

2. Answer each of the following short answer questions. You do **not** need to show your work.

(a) (10 points) A 2019 Toyota RAV4 vehicle gets 26 miles per gallon driving on city streets and 35 miles per gallon driving on highways. The fuel tank for the RAV4 holds 14.5 gallons. Suppose you drove 421 miles on a full fuel tank. Let x be the amount (in gallons) of fuel used for city driving and y be the amount (in gallons) used for highway driving. Set up the system of two equations that will solve for x and y . **DO NOT SOLVE THIS SYSTEM OF EQUATIONS.**

(b) (6 points) Let $A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -4 & 3 & 1 \\ -4 & 1 & 2 & 0 \\ 4 & 3 & 5 & -1 \end{pmatrix}$. Then the product AB is a $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$ matrix.

(c) (6 points) The sequence below is geometric. Determine the missing term.

$$3, \underline{\hspace{1cm}}, 192, 1536$$

(d) (8 points) The **demand** equation for a product is given by $p = 88 - 11q$. The **cost** equation for producing q units is $C = 4 + 14q$. Determine the **profit** function $P(q)$.

$$P(q) = \boxed{\hspace{10cm}}$$

3. Answer each of the following short answer questions. You do **not** need to show your work.

(a) (6 points) Solve the inequality below. Write your answer in interval notation.

$$\frac{5t + 1}{-2} > \frac{4 - 3t}{-2}$$

Answer:

(b) (6 points) Determine the composition $(f \circ g)(x)$ for $f(x) = x^2 + x$ and $g(x) = e^x$.

$$(f \circ g)(x) =$$

(c) (6 points) Give an equation of the line that passes through the point $(-2, 3)$ that is perpendicular to the line $y = 4x + 6$. Write the equation in either point-slope form or slope-intercept form.

Answer:

(d) (6 points) Determine A and B in the matrix equation $AX = B$, with A as the coefficient matrix of the following system of equations and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{cases} 2x + 3y + z = 7 \\ -2x + 5y = -3 \\ x + z = 0 \end{cases}$$

$$A = \begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix} \quad B = \begin{pmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{pmatrix}$$

4. Answer the following interest theory questions.

- (a) (10 points) Suppose \$2000 is deposited into an account that gains interest at a rate of 6% compounded continuously. At time t (in years), the account has \$3500. Find t , rounded to two decimal places. **Write the formula that you use to calculate t .**

$$t = \boxed{}$$

- (b) (10 points) Abe deposits \$400 into an investment fund at the end of every quarter. The fund gains interest at 5%, compounded quarterly. After 20 years, the account is worth X dollars. Find X , rounded to two decimal places. **Write the formula that you use to calculate X .**

$$X = \boxed{}$$

- (c) (10 points) You borrow \$20,000 which will be repaid over the next 5 years with monthly payments of Y . The interest rate charged on the loan is 6.6% compounded monthly. Find Y , rounded to two decimal places. **Write the formula that you use to calculate Y .**

$$Y = \boxed{}$$

5. Solve the equations. Separate multiple solutions with a comma. Show all of your work. **Solutions by calculator will receive no credit.**

(a) (12 points)

$$x + \sqrt{x + 7} = 5$$

$x =$

(b) (12 points)

$$\log_2(x - 3) + \log_2(x - 1) = 3$$

$x =$

(c) (12 points)

$$\log_x(12 - x) = 2$$

$x =$

6. (12 points) Solve the given nonlinear system of equations. Give your answers as ordered pairs of integers and/or fractions, and separate multiple solutions with commas. Show all of your work. **No credit will be given to calculator solutions.**

$$\begin{cases} x^2 + 3y^2 &= 4 \\ x + 2y &= 1 \end{cases}$$

$$(x, y) = \boxed{}$$

7. (12 points) You are given the following supply and demand equations:

$$\begin{cases} S(q) &= \frac{q}{40} + 10 \\ D(q) &= \frac{8000}{q} \end{cases}$$

Determine the equilibrium quantity, q . Show all of your work. **No credit will be given to calculator solutions.**

$$q = \boxed{}$$

8. (12 points) Let $f(x) = 2x^2 - 3x$. Determine the expression for $\frac{f(x+h) - f(x)}{h}$. You must simplify your result.

$$\frac{f(x+h) - f(x)}{h} =$$

9. (10 points) You are given that

$$\log x = 2$$

$$\log y = -1$$

$$\log z = 6$$

Compute

$$\log \left(\frac{x^{2.5} \cdot y^2}{z} \right)$$

$$\log \left(\frac{x^{2.5} \cdot y^2}{z} \right) =$$

10. Perform the indicated operations on the following matrices.

(a) (8 points) Let

$$A = \begin{bmatrix} 7 & 3 \\ 4 & -8 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ -2 & 8 \end{bmatrix}$$

Determine the product AB

$$AB = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

(b) (12 points) Use an augmented matrix, $[C|I]$, and elementary row operations to find the inverse of the following matrix:

$$C = \begin{bmatrix} -3 & 8 \\ 2 & -5 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

Scrap work

Some Useful Formulas

$$S = P(1 + r)^n$$

$$P = S(1 + r)^{-n}$$

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

$$S = Pe^{rt}$$

$$P = Se^{-rt}$$

$$r_e = e^r - 1$$

$$A = Ra_{\overline{n}|r} = R \frac{1 - (1 + r)^{-n}}{r}$$

$$R = \frac{A}{a_{\overline{n}|r}} = A \frac{r}{1 - (1 + r)^{-n}}$$

$$S = Rs_{\overline{n}|r} = R \frac{(1 + r)^n - 1}{r}$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r}$$

$$\sum_{i=1}^k ar^{i-1} = \frac{a(1 - r^k)}{1 - r}$$

$$\text{Int}_k = R \cdot [1 - (1 + r)^{-n+k-1}]$$

$$\text{Prin}_k = R \cdot (1 + r)^{-n+k-1}$$