Instructions:

- You have **1 hour and 45 minutes** to complete this exam. It consists of 12 questions on 11 pages including this cover sheet and is worth a total of 200 points. The value of each question is listed below and with each question. Partial credit might not be awarded on some questions.

- You may not use any books, notes or self-supplied scratch paper during this exam.

- Calculators are permitted EXCEPT those calculators that have symbolic algebra or calculus capabilities. In particular, the following calculators and their upgrades are not permitted: TI-89, TI-92, and HP-49. In addition, neither PDAs, laptops nor cell phones are permitted.

- Make sure to read each question carefully.

- Please **write clearly** and make sure to **justify your answers**. Correct answers with no supporting work may receive no credit. If you find a solution to a problem using a graph from your calculator (where appropriate), you need to sketch that graph and label all relevant information.

- Unless otherwise specified, make sure your answers are in **exact form** (i.e. not decimal approximations).

- Make sure to **circle** your answers.

Some useful formulas:

\[
A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = Pe^{rt} \quad A = P(a)^t \quad A = P(1 + r)^t
\]

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Score:
(1). Solve the following inequalities and write your solution in interval notation.

(a) (16 points)

\[ \frac{7}{6-x} \leq \frac{5}{9-x} \]

(b) (12 points)

\[ 3|8-y| + 3 \geq 18 \]
(2). The following questions concern properties of logarithms:

(a) (8 points) Expand the following logarithmic expression as much as possible:

$$\ln \left( \frac{(3 - x)^5(x^2 - 9)}{\sqrt{4 + x}} \right)$$

(b) (8 points) Write the following expression as a single logarithm:

$$\frac{1}{5} \ln(x - 5) - 3[\ln(4 - x) + \ln(25 - x^2)]$$
(3). Suppose 

\[ f(x) = 10^{x-3} + 6 \]

(a) (3 points) What are the domain and range of \( f(x) \) in interval notation?

\[
\text{Domain of } f(x): \quad \text{Range of } f(x): \quad \text{______________________} \quad \text{______________________}
\]

(b) (8 points) Find the inverse function \( f^{-1}(x) \)

(c) (5 points) What are the domain and range of \( f^{-1}(x) \) in interval notation?

\[
\text{Domain of } f^{-1}(x): \quad \text{Range of } f^{-1}(x): \quad \text{______________________} \quad \text{______________________}
\]
(4). Suppose that $f(x) = 3x^2 - 12x + 11$

(a) (7 points) Write $f(x)$ in vertex form by completing the square.

(b) (2 points) Identify the vertex.

(5). (12 points) Find the average rate of change of $f(x) = \frac{3}{x}$ from $x = a$ to $x = a + h$. Simplify your answer as much as possible.
(6). (12 points) Use long division to divide the following polynomials. Note: synthetic division will not work.

\[
\frac{2x^3 - 4x^2 + 3x - 5}{x^2 + 2}
\]

Quotient polynomial = \( Q(x) = \) __________

Remainder polynomial = \( R(x) = \) __________
(7). (10 points) Suppose the population of a city has been growing at a rate of 2.5% per year for each of the last 8 years. If the current population is 220,000, estimate the population 8 years ago. Round your answer to the nearest person.

(8). (10 points) A pickup truck gets 14 mpg in the city and 26 mpg on the highway. Suppose that the driver drives 250 miles using 11 gal of gas. Setup, but DO NOT SOLVE, a system of equations to find how many city miles and how many highway miles that the truck was driven. Make sure to clearly identify your variables.
(9). Solve the given linear system for the given variables:

(a) (9 points)

\[-8r + 6s = 2\]
\[5r + 2s = 16\]

(b) (12 points)

\[-4c + 5v + 3z = -20\]
\[-3c - 2v + 12z = 51\]
\[3c - 4v - 3z = 12\]
The following questions concern radioactive decay:

(a) (12 points) A sample 55mg of Cs-137 decays to 30mg in 26.4 years. Find a function of the form $Q(t) = Q_0e^{-kt}$ which models the amount $Q(t)$ of this sample of Cs-137 left after $t$ years. Round $k$ to four decimal places.

(b) (12 points) Suppose that $Q(t) = 25e^{-0.5924t}$ models the amount $Q(t)$, in mg, of a highly radioactive sample remaining after $t$ minutes. How long will it take until there is only 5mg remaining? Round your answer to four decimal places.
(11). Solve for $x$:

(a) (12 points)

\[ e^{2x} - 3e^x = 10 \]

(b) (12 points)

\[ \ln x + \ln (x - 7) = \ln (9x - 55) \]
(12). Consider the rational function $r$ whose graph is given below.

(a) (2 points) Find the $y$-intercept(s). Indicate if there are none.

(b) (4 points) Find the $x$-intercept(s). Indicate if there are none.

(c) (4 points) Find the equations of any vertical asymptotes.

(d) (2 points) Find the equation of the horizontal asymptote (you may assume it has one).

(e) (6 points) Write an equation for the rational function $r$:

$$r(x) = \text{__________________________}$$