Math 1151 PRACTICE Midterm 2

Name: _____

OSU name.#: _____

Recitation Instructor: _____

Recitation Time: _____

Instructions.

- **Show all relevant work** to receive full credit.
  Incorrect answers with substantially correct work may receive partial credit.
  Unsupported answers may receive no credit.

- Some parts of some problems are **multiple choice**. Circle *exactly one* choice.
  Ambiguous markings may receive no credit.

- Give **exact** answers unless instructed to do otherwise.

- **No calculators, phones, or other devices may be used** during the exam.

  Do not have these devices out!

- No notes or references are permitted.

- The allotted time for this exam is **55 minutes**.

- The exam consists of 7 problems starting on Page 2 and ending on Page 7. Check that your exam is complete before you begin.

Problem 1 [16 points]	
Problem 2 [20 points]	
Problem 3 [20 points]	
Problem 4 [22 points]	
Problem 5 [22 points]	
Total [100 points]	

1. (16 pts) Let f be the function with domain $[0, 6]$ given by the graph below.
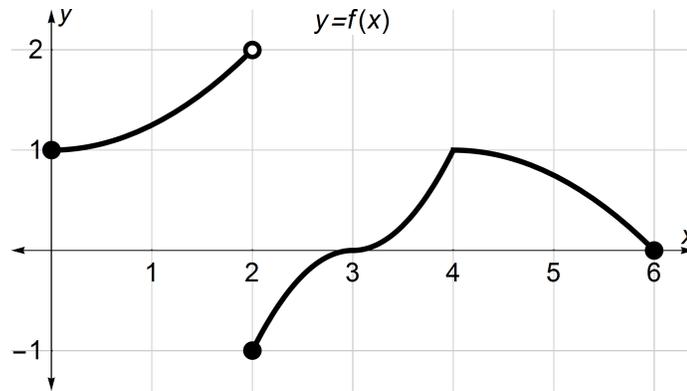
Use the graph of f to answer the following questions about f.

(a) List the **critical point(s)** of f on the interval $(0, 6)$. Write "none" if appropriate.

(b) List the **inflection point(s)** of f on the interval $(0, 6)$. Write "none" if appropriate.

Use the graph of f to answer the following questions about the **derivative f′** of f.

(c) List the **intervals(s)** on which f' is **negative**. Write "none" if appropriate.

(d) List the **intervals(s)** on which f' is **decreasing**. Write "none" if appropriate.

(e) List the **x-coordinates(s)** at which f' has a **local minimum**. Write "none" if appropriate.

2. (20 pts)

An object begins moving along a vertical line at time $t = 0\,\text{s}$ and stops at time $t = 6\,\text{s}$. Its height above the ground at time t is given by $h(t)$ in the provided graph, where h is measured in meters. Use the graph to answer the following questions.
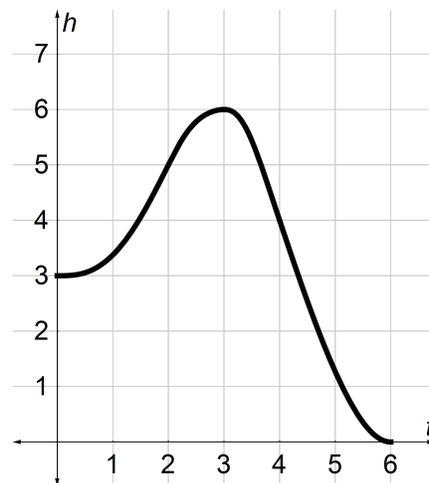
(a) Compute the average velocity of the object on the time interval $[0, 6]$. Show your work.

(b) **Circle the choice** that best describes the shape of the graph on the time interval $(5, 6)$.

  i. increasing and concave up

  ii. increasing and concave down

  iii. decreasing and concave up

  iv. decreasing and concave down

(c) At what time is the object farthest away from the ground? What is the velocity at that time?

  time =   velocity =

(d) The **maximum velocity** attained on $(0, 6)$ occurs at one of the following times. **Circle** the correct choice.

  i. 1 s   ii. 2 s   iii. 3 s   iv. 4 s   v. 5 s

(e) Estimate the value of the maximum velocity attained. The value is an integer.

(f) Let $v(t)$ be the velocity at time t. **Circle** the greatest value amongst the following choices.

  i. $v(2.5)$ ii. $v(2.6)$ iii. $v(2.7)$ iv. $v(2.8)$ v. $v(2.9)$

(g) List the time interval(s) on which the velocity is increasing. Use integer endpoints.

(h) The **maximum speed** attained on $(0, 6)$ occurs at one of the following times. **Circle** the correct choice.

  i. 1 s   ii. 2 s   iii. 3 s   iv. 4 s   v. 5 s

3. (20 pts) Let f be a differentiable one-to-one function on the interval $(0, 4)$ with the particular values of f and f' given in the table below, and let f^{-1} be the inverse function of f. Let g be the function given by the graph below.

x	f(x)	f'(x)
1	2	4
2	3	5
3	4	−2
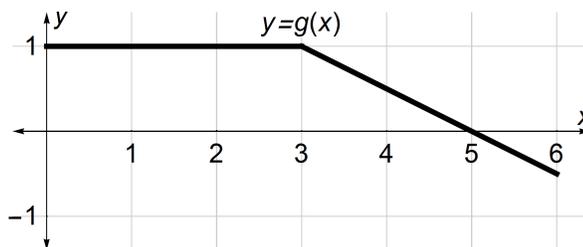
For (a), (b), and (c), **circle the correct choice.**

(a) $\lim\limits_{h \to 0} \dfrac{g(3 + h) - g(3)}{h}$

    i. $\frac{1}{2}$ iii. 5   v. −2   vii. none of the previous choices

    ii. $-\frac{1}{2}$ iv. 0   vi. does not exist

(b) $\dfrac{d}{dx}\left[g'(x + 2)f(x)\right]$ at $x = 3$

    i. $\frac{1}{2}$ iii. 1   v. −1   vii. none of the previous choices

    ii. $-\frac{1}{2}$ iv. 0   vi. does not exist

(c) $\dfrac{d}{dx}\left[f^{-1}(x)\right]$ at $x = 2$

    i. $-\frac{1}{2}$ iii. $\frac{1}{3}$ v. $\frac{1}{5}$ vii. none of the previous choices

    ii. $\frac{1}{2}$ iv. $\frac{1}{4}$ vi. does not exist

(d) Consider the curve given by $\quad xe^y + y = f(x) - 1.$

    Use **implicit differentiation** to compute $\frac{dy}{dx}$ at the point $(2, 0)$. Show your work.

4. (22 pts) The function f, its derivative f', and its second derivative f'' are given by the expressions below.

$$f(x) = \frac{48}{12 + (x+1)^2} \qquad f'(x) = \frac{-96(x+1)}{\left(12 + (x+1)^2\right)^2} \qquad f''(x) = \frac{288(x^2 + 2x - 3)}{\left(12 + (x+1)^2\right)^3}$$

(a) **Circle the correct statement.**

   i. f is even.   ii. f is odd.   iii. f is neither even nor odd.

(b) Give an **equation** for each horizontal asymptote of f. Write "none" if appropriate. Show your work.

Show work for parts (c), (d), (e), and (f) in the space below.

(c) List the interval(s) on which f is increasing. Write "none" if appropriate.

(d) List the interval(s) on which f is decreasing. Write "none" if appropriate.

(e) List the x-value(s) at which the local minima of f occur. Write "none" if appropriate.

(f) List the x-value(s) at which the local maxima of f occur. Write "none" if appropriate.

Show work for parts (g), (h), and (i) in the space below.

(g) List the interval(s) on which f is concave up. Write "none" if appropriate.

(h) List the interval(s) on which f is concave down. Write "none" if appropriate.

(i) List the x-coordinate(s) of the inflection points of f. Write "none" if appropriate.

(j) Sketch the graph of $y = f(x)$. Mark the locations of any local extrema and/or inflection points with filled-in circles. Your sketch should be consistent with your answers to (a)-(i).
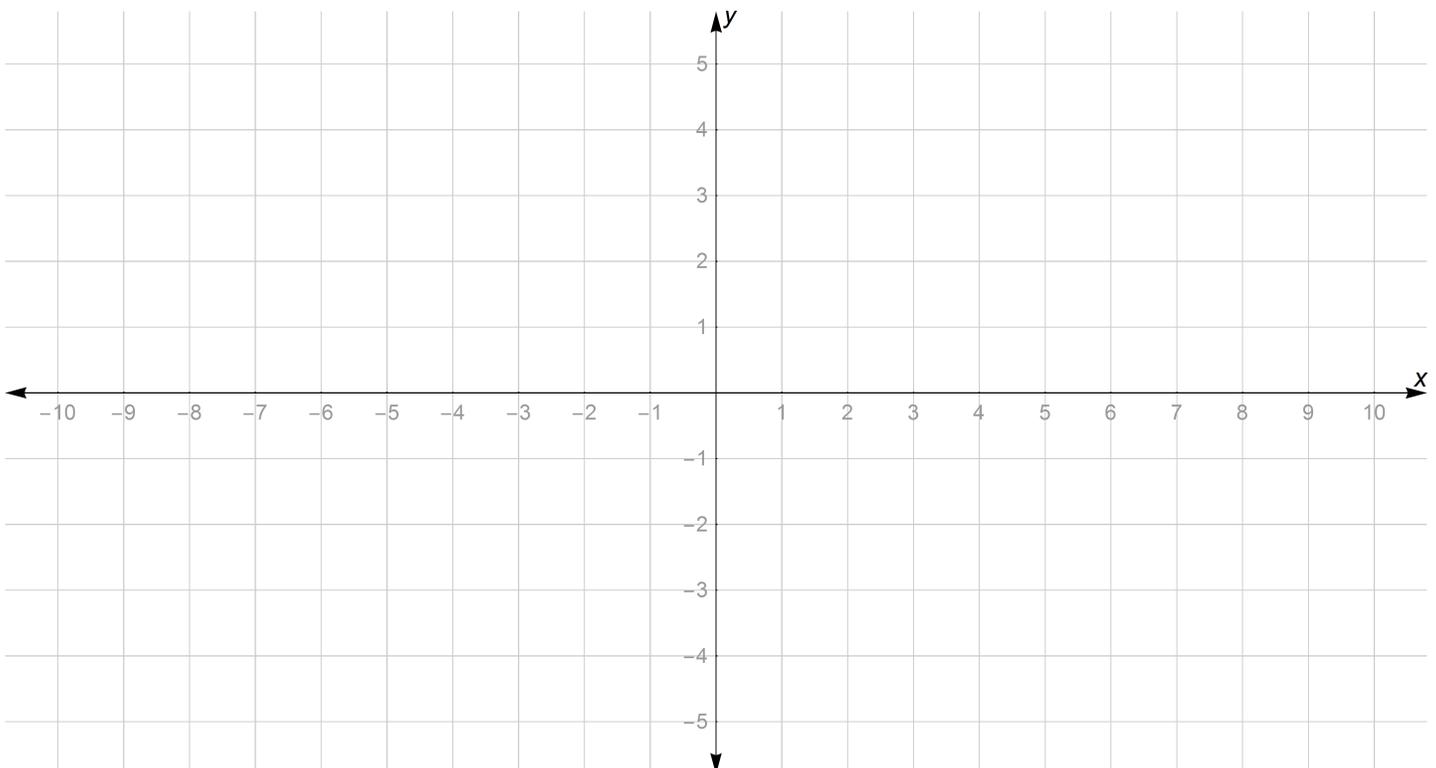
5. (22 pts) A window has the shape of a right triangle with base length 12 ft and height 6 ft. The curtain is being opened from left to right revealing the window as shown in the pictures below.
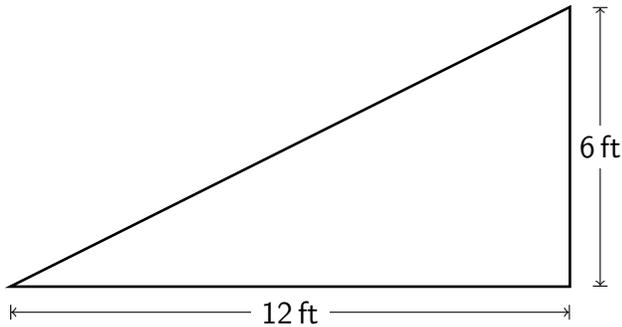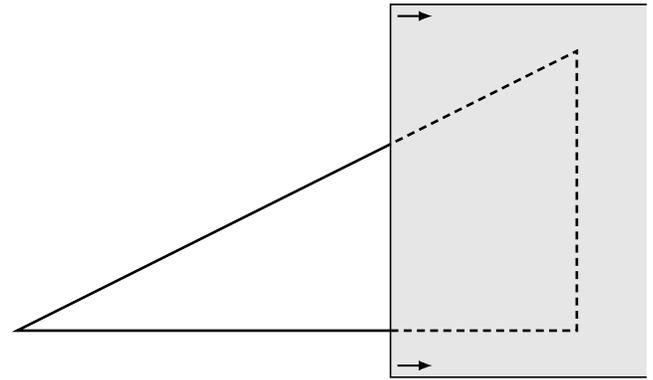
Figure 1: Window without curtain   **Figure 2: Window with curtain**

Let x represent the base length of the exposed portion of the window, and let y represent the height of the exposed portion of the window.

(a) Label x and y on Figure 2 above.

(b) Suppose that the curtain is opening at a rate of 4 ft/s. At what rate is the area A of the exposed portion of the window changing when the area of the exposed portion is 9 ft^2? Include appropriate units. Show your work.