

**Math 1151 Midterm 1**

Name: \_\_\_\_\_

January 30, 2018

OSU name.#: \_\_\_\_\_

Form A

Lecturer: \_\_\_\_\_

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Recitation Instructor: \_\_\_\_\_

Recitation Time: \_\_\_\_\_

**Instructions.**

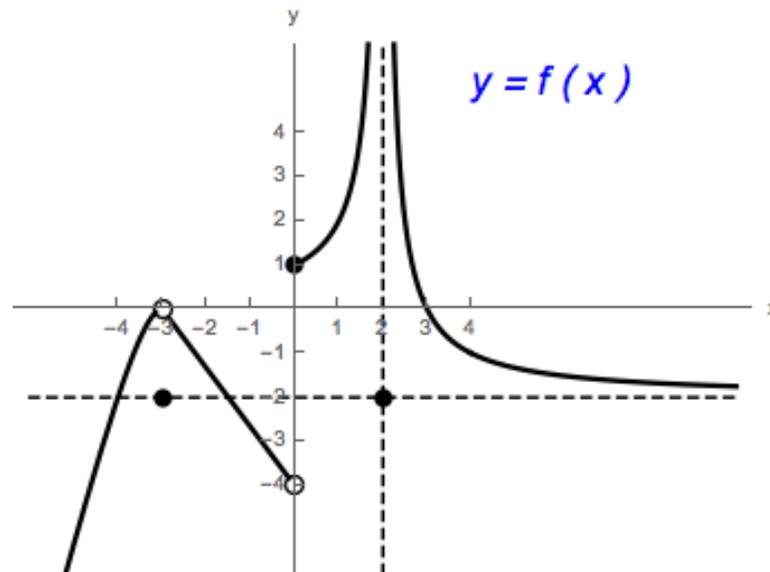
- **Show all relevant work** to receive full credit on Problems 2, 3, and 4. Incorrect answers with substantially correct work may receive partial credit. **Unsupported answers may receive no credit.**

You do not need to show work for Problem 1.

- Give **exact** answers unless instructed to do otherwise.
- **No calculators, phones, or other devices may be used** during the exam. Do not have these devices out!
- No notes or references are permitted.
- The allotted time for this exam is **55 minutes**.
- The exam consists of 4 problems starting on Page 2 and ending on Page 7. Check that your exam is complete before you begin.

<b>Problem 1</b> [18 points]	
<b>Problem 2</b> [18 points]	
<b>Problem 3</b> [18 points]	
<b>Problem 4</b> [21 points]	
<b>Total</b> [75 points]	

1. (18 pts) The graph of a function  $f$ , defined on  $(-\infty, +\infty)$ , is given in the figure below.



Use the graph of  $f$  to answer questions (a)-(f) below.

- (a) Determine the value or write "does not exist".

i.  $f(-3) =$

ii.  $f(0) =$

- (b) Determine the limit or write "does not exist" (DNE). Write "does not exist" only if a limit does not exist and is not  $+\infty$  or  $-\infty$ .

i.  $\lim_{x \rightarrow -3} f(x) =$

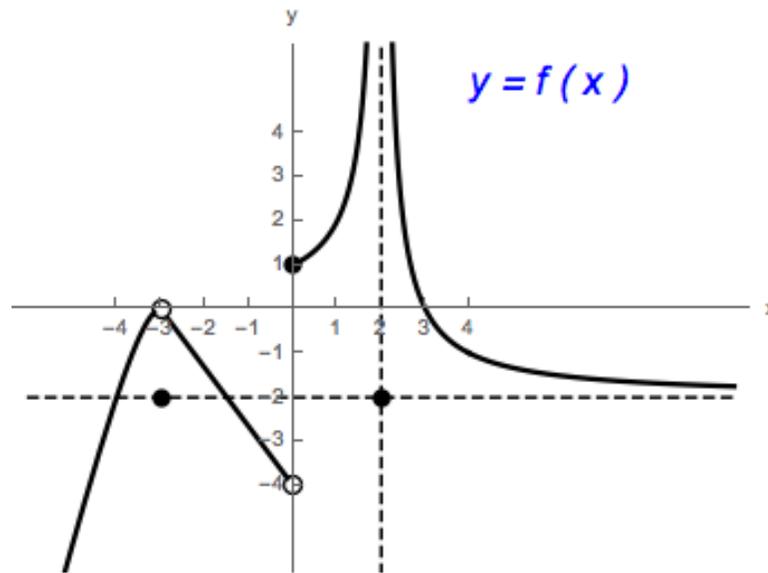
iv.  $\lim_{x \rightarrow 2} f(x) =$

ii.  $\lim_{x \rightarrow 0^-} f(x) =$

v.  $\lim_{x \rightarrow -\infty} f(x) =$

iii.  $\lim_{x \rightarrow 0} f(x) =$

vi.  $\lim_{x \rightarrow +\infty} f(x) =$



(c) MULTIPLE CHOICE! Circle ONLY one answer!

Find a number  $a$  for which the following statement is true (the function  $f$  is shown above):

STATEMENT: The function  $f$  is **continuous** at  $a$ , and  $\lim_{x \rightarrow a} f(x) = 0$ .

i.  $a = -3$

iii.  $a = 0$

v.  $a = 3$

ii.  $a = -2$

iv.  $a = 2$

vi. Such a number does not exist.

(d) Determine the (largest) **intervals of continuity** of  $f$ . Use interval notation to write your answer.

(e) Write the **equation(s)** of all **vertical asymptotes** of  $f$ . Write "none" if appropriate.

(f) Write the **equation(s)** of all **horizontal asymptotes** of  $f$ . Write "none" if appropriate.

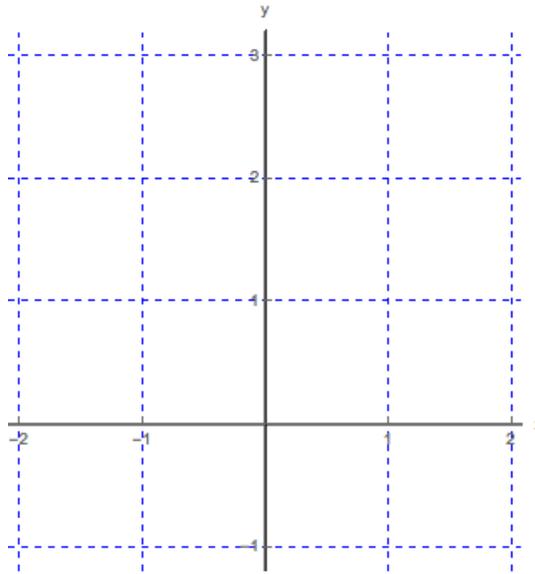
2. (18 pts) Consider three functions,  $g$ ,  $f$ , and  $h$ , defined on the interval  $(-2, 2)$ .

Given that

$$g(x) = \cos(\pi x), \quad h(x) = |x| + 1, \quad \text{and} \quad g(x) \leq f(x) \leq h(x), \quad -2 < x < 2,$$

answer the questions below.

- (a) Sketch and label the graph of  $g$  and  $h$ , and a possible graph of  $f$  in the figure below.



- (b) Use the Squeeze Theorem to evaluate the limit:  $\lim_{x \rightarrow 0} f(x)$ . Show your work.

WORK:

ANSWER:  $\lim_{x \rightarrow 0} f(x) =$

- (c) Write the **form** of the limit and evaluate the limit.

Write “does not exist” only if the limit does not exist and is not  $+\infty$  or  $-\infty$ .

Show your work. Justify your answer.

$$\lim_{x \rightarrow 0^+} \frac{g(x)}{x^2 - 5x} = \lim_{x \rightarrow 0^+} \frac{\cos(\pi x)}{x^2 - 5x} =$$

FORM:

3. (18 pts) Show your work.

Let  $f$  be a function defined by

$$f(x) = \frac{12}{x+2}$$

(a) Write the **form** of the limit and evaluate the limit. Do not use L'Hôpital's Rule. Show your work.

$$\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} =$$

FORM:

(b) Find the (natural) domain of  $f$ . Write your answer in interval notation.

(c) Let  $f^{-1}$  be an inverse of  $f$ . Find an expression for  $f^{-1}(x)$ , for  $x$  in the domain of  $f^{-1}$ .

$$f^{-1}(x) =$$

(d) Find the value or say "does not exist" (DNE).

i.  $f^{-1}(f(\pi)) =$

ii.  $f(f^{-1}(0)) =$

4. (21 pts) Let  $g$  be the function given by

$$g(x) = \begin{cases} \frac{6-x}{2+x} & \text{if } x < 2, x \neq -2 \\ \frac{\sqrt{x^2 - x - 1}}{x-1} & \text{if } x \geq 2. \end{cases}$$

(a) (6 pts) Use the **definition of continuity** to determine whether the function  $g$  is continuous at 2. Show your work.

(b) (3 pts) Evaluate the limit. Justify your answer. Show your work.  
Write “does not exist” only if the limit does not exist and is not  $+\infty$  or  $-\infty$ .

$$\lim_{x \rightarrow -2^+} g(x) =$$

(c) (2 pts) Write the **equation(s)** of all **vertical asymptotes** of  $g$ . Write “none” if appropriate.

(d) (2 pts) Determine the (largest) **intervals of continuity** of  $g$ .  
Use interval notation to write your answer.

Let  $g$  be the function given by

$$g(x) = \begin{cases} \frac{6-x}{2+x} & \text{if } x < 2, x \neq -2 \\ \frac{\sqrt{x^2 - x - 1}}{x-1} & \text{if } x \geq 2. \end{cases}$$

(e) (6 pts) Evaluate each limit. Do not use L'Hôpital's Rule.

Write "does not exist" only if the limit does not exist and is not  $+\infty$  or  $-\infty$ . Show your work.

i.  $\lim_{x \rightarrow -\infty} g(x) =$

ii.  $\lim_{x \rightarrow +\infty} g(x) =$

(f) (2 pts) Write the **equation(s)** of all **horizontal asymptotes** of  $g$ .

Write "none" if appropriate.

B L A N K P A G E