Instructions.

- **Show all relevant work** to receive full credit on Problems 1, 2 I, 2 II(a),(b), 4, and 5. Incorrect answers with substantially correct work may receive partial credit. **Unsupported answers may receive no credit.**
- You don’t have to show work in Problem 2 II(c) and Problem 3. Some parts in Problem 2 and Problem 3 are **multiple choice.** Circle exactly one choice. **Ambiguous markings may receive no credit.**
- Give exact answers unless instructed to do otherwise.
- **No calculators, phones, or other devices may be used** during the exam. Do not have these devices out!
- No notes or references are permitted.
- The allotted time for this exam is **55 minutes.**
- The exam consists of 5 problems starting on Page 2 and ending on Page 8. Check that your exam is complete before you begin.

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1. (38 pts) Show your work!

(a) Consider the limit: \( \lim_{x \to 0^+} \frac{1}{x} [1 - \sin x]^x \).

i. Write the **form** of the limit.

**FORM:**

ii. Evaluate the limit. If the limit does not exist, write "DNE". You may use L'Hôpital’s Rule.

(b) Let \( f \) be a function defined by \( f(x) = e^{-x} \).

i. Find \( L(x) \), the linear approximation to \( f \) at \( a = 0 \).

ii. Use \( L(x) \) from part (i) to estimate the value \( e^{-0.4} \).

iii. State whether the estimate in part(ii) is an underestimate or an overestimate. Explain.
(c) The velocity, $v(t)$, of an object moving along a straight line at the time $t$ is given by

$$v(t) = t + \sin(\pi t), \quad 0 \leq t \leq 2.$$ 

Here, $t$ is measured in seconds and $v(t)$ in m/s.

Given that $s(0) = 0$, find the position, $s(t)$, of the object at the time $t$, $0 \leq t \leq 2$.

(d) Evaluate the integral. (Hint: Use symmetry.)

$$\int_{-3}^{3} \left( x^2 + \frac{6x^5}{4 + x^6} \right) \, dx$$
2. (18 pts) The function $f$ is continuous and piecewise linear on $[0, 12]$. The graph of $f$ is shown below.

Use the graph of $f$ to answer the following questions.

PART I

(a) Show your work!

Use geometry to evaluate the integral $\int_1^7 f(x) \, dx$. 

(b) Show your work! (HINT: Use the result in part (a).)

Find $\bar{f}$, the average value of $f$ on $[1, 7]$. 

(c) Show your work! (HINT: Use the result in part (a).)

Evaluate the integral $\int_1^7 (2f(x) + 1) \, dx$. 

PART II

(a) Illustrate the right Riemann sum of \( f \) on \([0, 12]\) for \( n = 3 \)

by sketching appropriate rectangles on the figure below.

![Diagram of a function and rectangles]

(b) Compute the right Riemann sum of \( f \) on \([0, 12]\) for \( n = 3 \). Show your work.

(c) Find the expression for the right Riemann sum of \( f \) on \([0, 12]\), for any positive integer \( n \).

Circle the correct choice.

\[
\begin{align*}
\text{i. } & \sum_{k=1}^{n} f \left( \frac{k}{n} \right) \\
\text{ii. } & \sum_{k=1}^{n} f \left( \frac{k}{n} \right) \frac{12}{n} \\
\text{iii. } & \sum_{k=1}^{n} f \left( \frac{12k}{n} \right) \\
\text{iv. } & \sum_{k=1}^{n} f \left( \frac{12k}{n} \right) \frac{12}{n} \\
\text{v. } & \sum_{k=0}^{n-1} f \left( \frac{12k}{n} \right) \frac{12}{n} \\
\text{vi. } & \text{No previous choice is true.}
\end{align*}
\]
3. (12 pts) A function $f$ is continuous on $[0, 12]$. The graph of $f$ is shown below.

Let $A(x) = \int_{0}^{x} f(t) \, dt$ for $0 \leq x \leq 12$.

Use the graph of $f$ to answer the following questions about the function $A$.

(a) Find the values.

i. $A(3) =$

ii. $A'(3) =$

(b) Circle the correct statement about the values $A(4)$ and $A(5)$.

i. $A(4) < A(5)$

ii. $A(4) = A(5)$

iii. $A(5) < A(4)$

iv. No previous choice is true.

(c) Complete the sentence.

The function $A$ has an absolute minimum value at $x =$ _____.

4. (18 pts) A rectangle is constructed in the first quadrant with one side on the x-axis, one side on the y-axis, and the vertex opposite the origin on the curve \( y = \sqrt{9 - x} \) (see figure).

Find the area of the largest such rectangle.

Solve the problem by following the steps indicated below.

(a) Label the picture.

(b) Express the objective function (the function to be optimized) in terms of a single variable, and state its domain.

(c) Use methods of calculus to solve the problem. Show your work and justify your answer.
5. (14 pts) Sketch the graph of a function $f$ satisfying all of the following conditions:

(a) Domain of $f = (-\infty, +\infty)$; 
(b) $f$ is continuous on its domain; 
(c) $f$ is even; 
(d) $f$ is not differentiable at $x = 4$; 
(e) $\lim_{x \to +\infty} f(x) = -6$; 
(f) $f(0) = 3$; 
(g) $f'(x) > 0$ on $(0, 4)$; 
(h) $f'(x) < 0$ on $(4, +\infty)$; 
(i) $f''(x) > 0$ on $(0, 4)$ and $(8, +\infty)$; 
(j) $f''(x) < 0$ on $(4, 8)$;