Name: ______________________

OSU name.#: ______________________

Lecturer: ______________________

Recitation Instructor: ______________________

Recitation Time: ______________________

Instructions.

- **Show all relevant work** to receive full credit on Problems 1, 2 I, 2 II(a),(b), 4, and 5.
  Incorrect answers with substantially correct work may receive partial credit.
  Unsupported answers may receive no credit.
- You don't have to show work in Problem 2 II(c) and Problem 3.
  Some parts in Problem 2 and Problem 3 are **multiple choice**.
  Circle *exactly one* choice.
  Ambiguous markings may receive no credit.
- Give **exact** answers unless instructed to do otherwise.
- **No calculators, phones, or other devices may be used** during the exam.
  Do not have these devices out!
- No notes or references are permitted.
- The allotted time for this exam is **55 minutes**.
- The exam consists of 5 problems starting on Page 2 and ending on Page 8.
  Check that your exam is complete before you begin.

Problem 1 [38 points]	
Problem 2 [18 points]	
Problem 3 [12 points]	
Problem 4 [18 points]	
Problem 5 [14 points]	
Total [100 points]	

1. (38 pts) Show your work!

   (a) Consider the limit: $\lim_{x\to 0^+} (1 - \sin x)^{\frac{1}{x}}$.

      i. Write the **form** of the limit.

         FORM:

      ii. Evaluate the limit. If the limit does not exist, write "DNE". You may use L'Hôpital's Rule.

   (b) Let f be a function defined by $f(x) = e^{-x}$.

      i. Find $L(x)$, the linear approximation to f at $a = 0$.

      ii. Use $L(x)$ from part (i) to estimate the value $e^{-0.4}$.

      iii. State whether the estimate in part(ii) is an underestimate or an overestimate. Explain.

(c) The velocity, $v(t)$, of an object moving along a straight line at the time t is given by

$$v(t) = t + \sin(\pi t), \qquad 0 \leq t \leq 2.$$

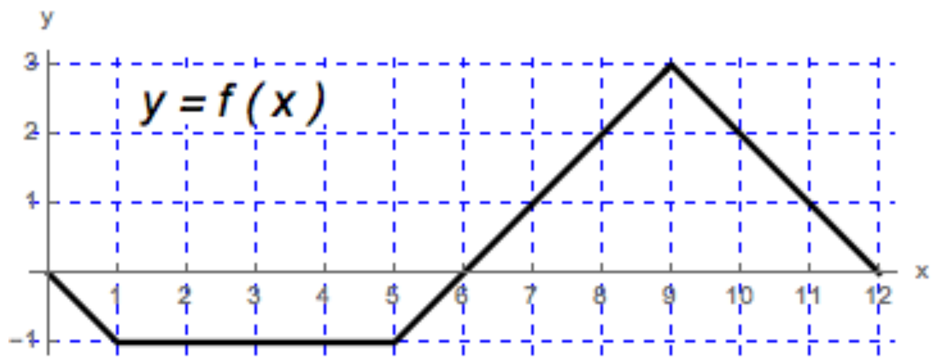
Here, t is measured in seconds and $v(t)$ in m/s.

Given that $s(0) = 0$, find the position, $s(t)$, of the object at the time t, $0 \leq t \leq 2$.

(d) Evaluate the integral. (Hint: Use symmetry.)

$$\int_{-3}^{3} \left(x^2 + \frac{6x^5}{4 + x^6} \right) \mathrm{d}x$$

2. (18 pts) The function f is continuous and piecewise linear on $[0, 12]$. The graph of f is shown below.



Use the graph of f to answer the following questions.

PART I

(a) Show your work!

Use geometry to evaluate the integral $\displaystyle\int_1^7 f(x)\,dx$.

(b) Show your work! (HINT: Use the result in part (a).)

Find $\bar{f}$, the average value of f on $[1, 7]$.

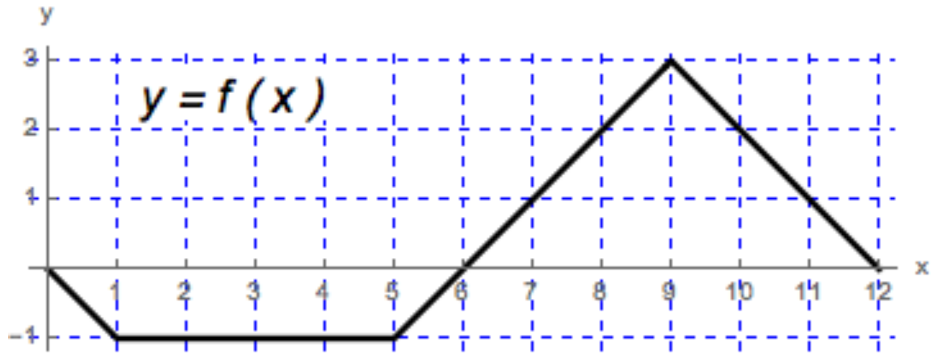
(c) Show your work! (HINT: Use the result in part (a).)

Evaluate the integral $\displaystyle\int_1^7 (2f(x) + 1)\,dx$.

PART II

(a) Illustrate the **right** Riemann sum of f on $[0,12]$ for $n=3$

by sketching appropriate rectangles on the figure below.



(b) Compute the **right** Riemann sum of f on $[0,12]$ for $n=3$. Show your work.

(c) Find the expression for the **right** Riemann sum of f on $[0,12]$, for any positive integer n.

Circle the correct choice.

i. $\displaystyle\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$

ii. $\displaystyle\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\frac{12}{n}$

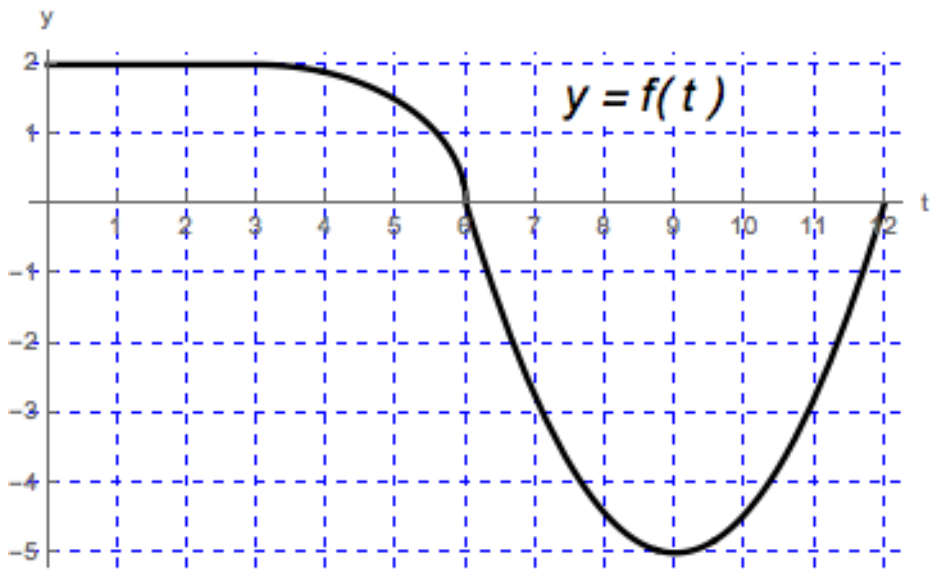
iii. $\displaystyle\sum_{k=1}^{n} f\left(\frac{12k}{n}\right)$

iv. $\displaystyle\sum_{k=1}^{n} f\left(\frac{12k}{n}\right)\frac{12}{n}$

v. $\displaystyle\sum_{k=0}^{n-1} f\left(\frac{12k}{n}\right)\frac{12}{n}$

vi. No previous choice is true.

3. (12 pts) A function f is continuous on $[0, 12]$. The graph of f is shown below.



Let $A(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 12$.

Use the graph of f to answer the following questions about the function A.

(a) Find the values.

i. $A(3) =$

ii. $A'(3) =$

(b) Circle the correct statement about the values $A(4)$ and $A(5)$.

i. $A(4) < A(5)$

iii. $A(5) < A(4)$

ii. $A(4) = A(5)$

iv. No previous choice is true.

(c) Complete the sentence.

The function A has an absolute minimum value at x= ____.

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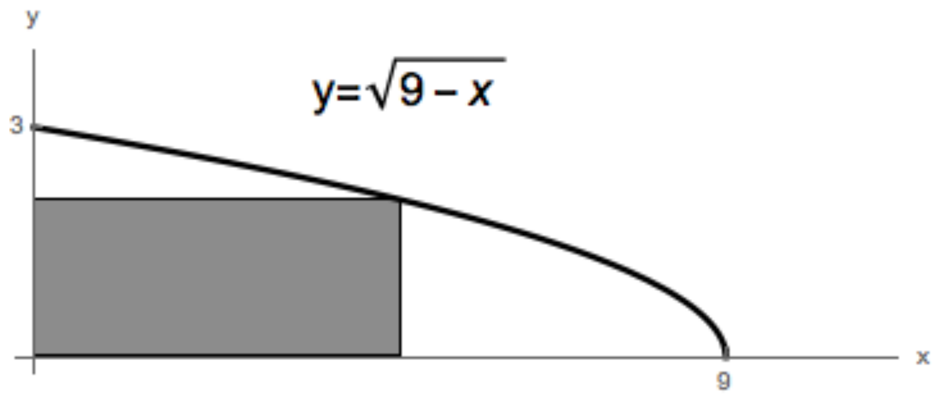
4. (18 pts) A **rectangle** is constructed in the **first quadrant** with one side on the x-axis, one side on the y-axis, and the vertex opposite the origin on the curve $y = \sqrt{9 - x}$ (see figure).

   Find the **area** of the **largest** such rectangle.

   Solve the problem by following the steps indicated below.

   (a) Label the picture.

   (b) Express the objective function (the function to be optimized) in terms of a single variable, and state its domain.

   (c) Use methods of calculus to solve the problem. Show your work and justify your answer.

5. (14 pts) Sketch the graph of a function f satisfying **all** of the following conditions:

(a) Domain of $f = (-\infty, +\infty)$;

(b) f is continuous on its domain;

(c) f is **even**;

(d) f is **not differentiable** at $x = 4$;

(e) $\lim_{x\to+\infty} f(x) = -6$;

(f) $f(0) = 3$;

(g) $f'(x) > 0$ on $(0, 4)$;

(h) $f'(x) < 0$ on $(4, +\infty)$;

(i) $f''(x) > 0$ on $(0, 4)$ and $(8, +\infty)$.

(j) $f''(x) < 0$ on $(4, 8)$;