Math 1151 Midterm 3

April 18, 2017

Form A

Name: _____

OSU name.#: _____

Lecturer: _____

Recitation Instructor: _____

Recitation Time: _____

Instructions.

- **Show all relevant work** to receive full credit on Problems 1, 2, 3, 5, and 6.
  Incorrect answers with substantially correct work may receive partial credit.
  Unsupported answers may receive no credit.

- Problem 4 is **multiple choice**. Circle *exactly one* choice.
  Ambiguous markings may receive no credit.

- Give **exact** answers unless instructed to do otherwise.

- **No calculators, phones, or other devices may be used** during the exam.
  Do not have these devices out!

- No notes or references are permitted.

- The allotted time for this exam is **55 minutes**.

- The exam consists of 6 problems starting on Page 2 and ending on Page 7. Page 8 is blank.
  Check that your exam is complete before you begin.

Problem 1 [16 points]	
Problem 2 [24 points]	
Problem 3 [16 points]	
Problem 4 [12 points]	
Problem 5 [14 points]	
Problem 6 [18 points]	
Total [100 points]	

1. (16 pts) Sketch the graph of a function f satisfying **all** of the following conditions:

   (a) Domain of $f = (-\infty, +\infty)$;

   (b) f is **even** ;

   (c) f is continuous on $(0, +\infty)$;

   (d) f is **not differentiable** at $x = 4$;

   (e) $\lim\limits_{x \to +\infty} f(x) = 7$, $\quad \lim\limits_{x \to 0^+} f(x) = +\infty$;

   (f) $f'(x) > 0$ on $(4, +\infty)$;

   (g) $f'(x) < 0$ on $(0, 4)$;

   (h) $f''(x) > 0$ on $(0, 2)$;

   (i) $f''(x) < 0$ on $(2, 4)$ and $(4, +\infty)$.
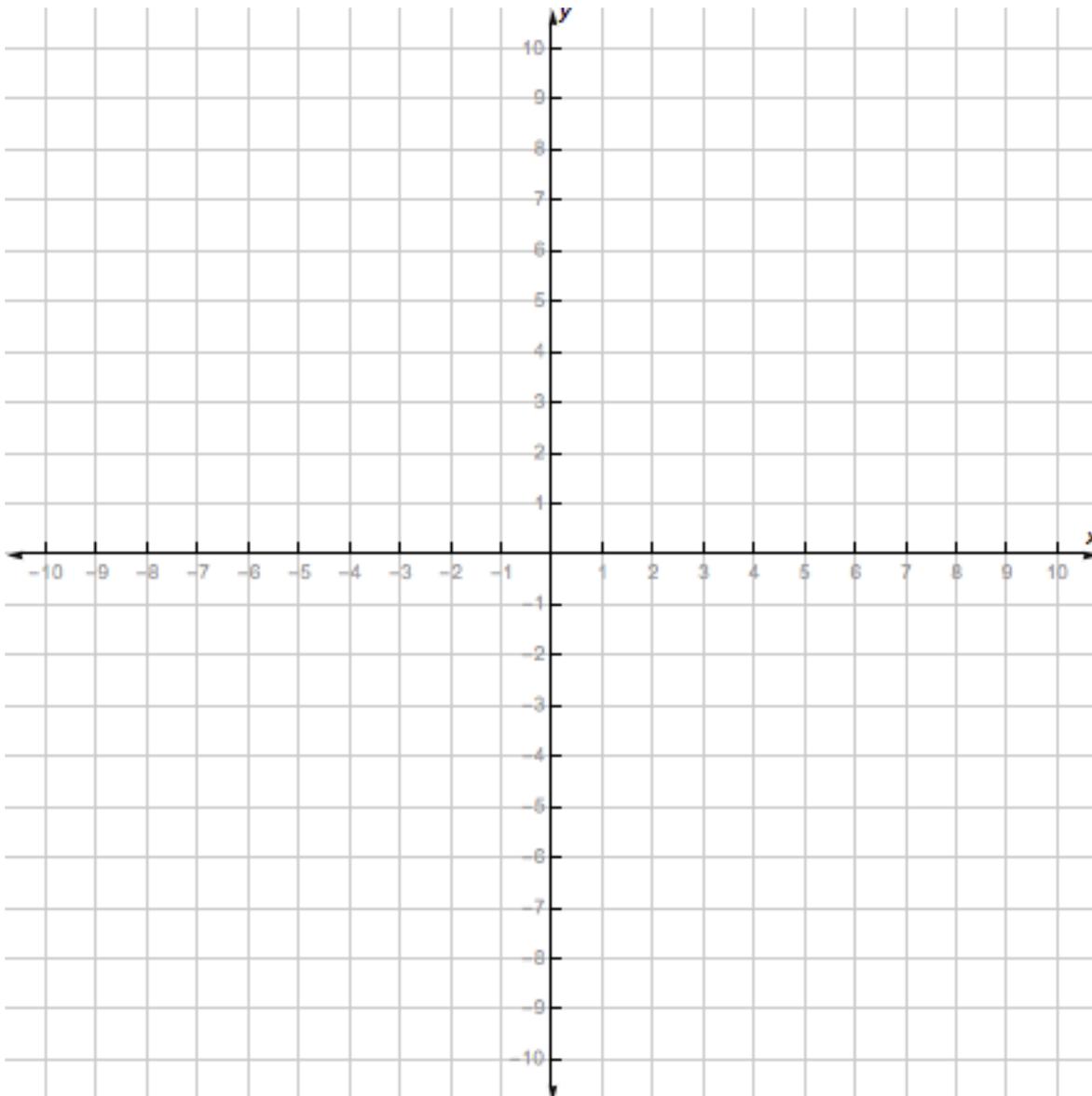
2. (24 pts) Show your work!

   (a) State the **form** of the limit, then evaluate the limit. If the limit does not exist, write "DNE".

   You may use L'Hôpital's Rule.

   $$\lim_{x \to -\infty} (x - 6)e^x$$

   (b) Evaluate the integral. (Hint: Use symmetry.)

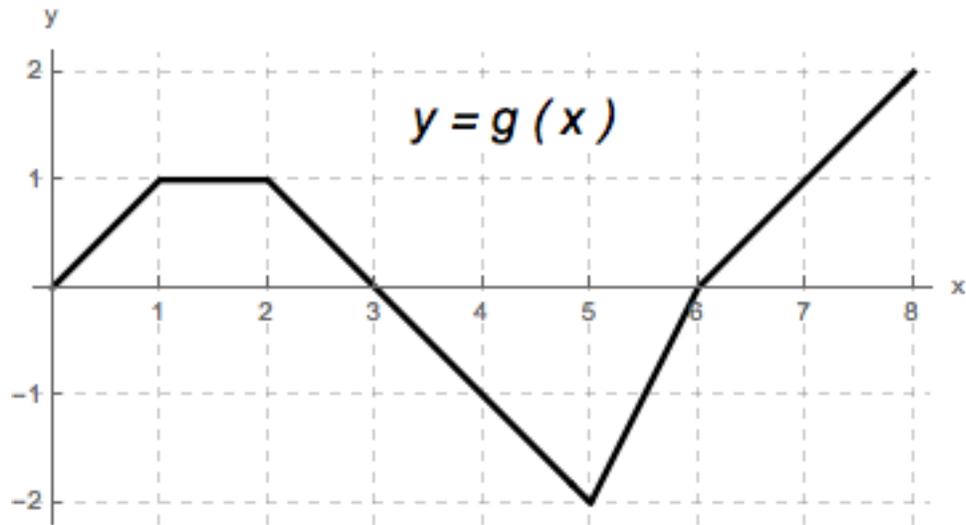 $$\int_{-1}^{1} \left(2x^{17} - sin(x) + \pi + \frac{1}{1 + x^2} \right) dx$$

   (c) Determine the derivative.

 $$\frac{d}{dx} \int_{0}^{ln(x)} \sqrt{4 - sin(t)} \, dt$$

   (d) Evaluate the sum.

   $$\sum_{k=1}^{3} \frac{6(k - 1)}{k + 1}$$

3. (16 pts) The function g is continuous and piecewise linear on $[0, 8]$. The graph of g is shown below.

$$y = g(x)$$

(a) Illustrate the **right** Riemann sum of g on $[0, 8]$ for $n = 4$

   by sketching appropriate rectangles on the figure above.

(b) Compute the **right** Riemann sum of g on $[0, 8]$ for $n = 4$. Show your work.

(c) Use geometry to evaluate the integral $\int_0^8 g(x)\, dx$. Show your work.

(d) Using your answer to part (c), evaluate the **limit of Riemann sums** on $[0, 8]$. Show your work!

$$\lim_{\Delta \to 0} \sum_{k=1}^{n} 2g(x_k^*)\Delta x_k$$

(e) Find $\bar{g}$, the average value of g on $[0, 8]$. You may use your answer to part (c).

4. (12 pts) The function f is continuous on $[0, 8]$. The graph of f is shown below.
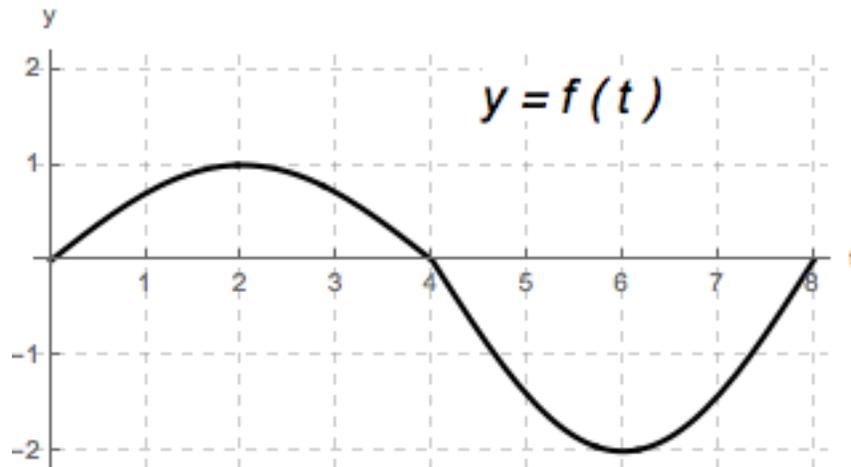
Let $A(x) = \displaystyle\int_0^x f(t) \, dt$ for $0 \le x \le 8$.

(a) Circle the correct statement below regarding the value of $A(4.2)$.

    i. $A(4.2) < 0$ iii. $A(4.2) > 0$

    ii. $A(4.2) = 0$ iv. No previous choice is true.

(b) Circle the correct statement below regarding the value of $A'(4.2)$.

    i. $A'(4.2) < 0$ iii. $A'(4.2) > 0$

    ii. $A'(4.2) = 0$ iv. No previous choice is true.

(c) Circle the choice that correctly completes the following sentence.

The function A attains its **maximum** value on $[0, 8]$ at _____.

    i. $x = 0$ iii. $x = 4$ v. $x = 8$

    ii. $x = 2$ iv. $x = 6$ vi. No previous choice is true.

(d) Circle the choice that correctly completes the following sentence.

The function A is **decreasing** and **concave DOWN** on the interval _____.

    i. $(0, 2)$ iii. $(4, 6)$ v. No such interval exists.
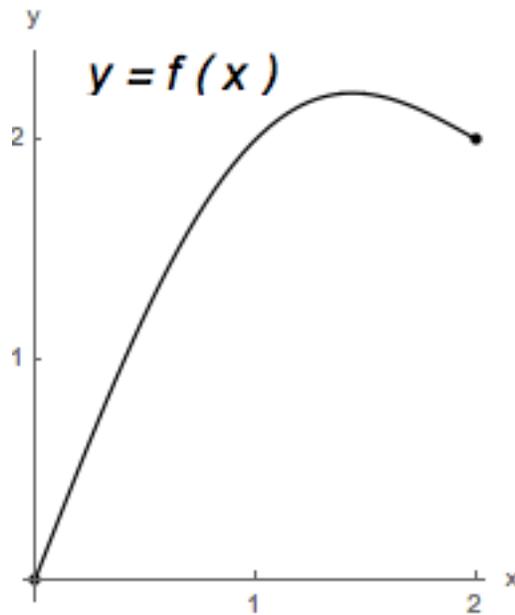
    ii. $(2, 4)$ iv. $(6, 8)$ vi. No previous choice is true.

5. (14 pts) Let $f(x) = x + \sin\left(\dfrac{\pi}{2}x\right)$. The graph of f on the interval $[0, 2]$ is given below.

(a) Determine whether the Mean Value Theorem applies to f on $[0, 2]$. If so, find the point(s) that are guaranteed to exist by the Mean Value Theorem.

(b) Find L(x), the linear approximation to f at a=1.
Sketch the graph of L in the figure above.

(c) Use the linearization from part (b) to estimate $f\left(\dfrac{6}{7}\right)$.

Is this an underestimate or overestimate? Explain.

6. (18 pts) An **isosceles** triangle has its vertex at the origin, its **base parallel to and above the x-axis**,

   and the vertices of its base on the parabola $y = f(x) = 27 - x^2$ (see figure).

   Find the **area** of the **largest** such triangle. Show your work and justify your answer.
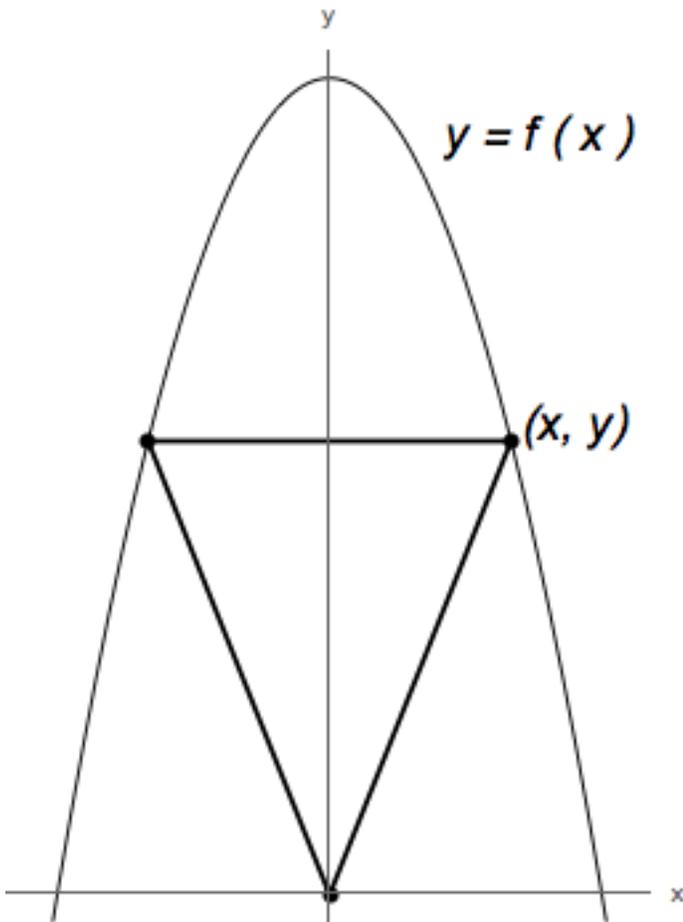
Solve the problem by following the steps indicated below.

(a) Label the picture.

(b) Express A, the area of the triangle, as a function of x (see figure). What is the domain of A?

   $A(x) =$

   Domain of $A =$

(c) Finish the problem.

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