Instructions.

- **Show all relevant work** to receive full credit on Problems 1, 2, 3, 4(a)-(c), 5, and 6. Incorrect answers with substantially correct work may receive partial credit. **Unsupported answers may receive no credit.**
- Problem 4(d) is **multiple choice**. Circle **exactly one** choice. **Ambiguous markings may receive no credit.**
- Give **exact** answers unless instructed to do otherwise.
- **No calculators, phones, or other devices may be used** during the exam. Do not have these devices out!
- No notes or references are permitted.
- The allotted time for this exam is **55 minutes**.
- The exam consists of 6 problems starting on Page 2 and ending on Page 7. Page 8 is blank. Check that your exam is complete before you begin.
1. (16 pts) Sketch the graph of a function $f$ satisfying all of the following conditions:

(a) $f$ is even;

(b) $f(0)=5$; $f$ is continuous, but not differentiable at $x = 0$;

(c) $\lim_{x \to +\infty} f(x) = 5$, $\lim_{x \to 6} f(x) = -\infty$;

(d) $f'(x) > 0$ on $(6, +\infty)$;

(e) $f'(x) < 0$ on $(0, 6)$;

(f) $f''(x) > 0$ on $(0, 3)$;

(g) $f''(x) < 0$ on $(3, 6)$ and $(6, +\infty)$. 

![Graph of function](image)
2. (18 pts) Show your work!
The function $f$ is continuous and piecewise linear on $[-5, 5]$. The graph of $f$ is shown twice below.

(a) Compute the right Riemann sum of $f$ on $[-5, 5]$ for $n = 2$.
Illustrate this Riemann sum by sketching appropriate rectangles in figure (A) above.

(b) Express the limit of Riemann sums of the function $f$ on the interval $[-5, 5]$ as a definite integral and use geometry to evaluate it. (You may use figure (B) for this computation.)

$$
\lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x
$$

(c) Use geometry and properties of definite integrals to evaluate the integral.
(You may use figure (B) for this computation).

$$
\int_{-5}^{5} (2f(x) - 1) \, dx
$$
3. (16 pts) Consider an object moving along a horizontal line. The velocity function \( v \) and the initial position \( s(0) \) of the object are given.

\[
\begin{cases}
  v(t) = t - \frac{\pi}{4} \sin\left( \frac{\pi}{4} t \right), & 0 \leq t \leq 8; \\
  s(0) = 0
\end{cases}
\]

(a) Find \( s(t) \), the position of the object at the time \( t \), for \( 0 \leq t \leq 8 \).

(b) Find the displacement of the object over the time interval \([0, 4]\).

(c) Express the displacement of the object over the time interval \([0, 4]\) as a definite integral.
4. (18 pts) Let $f$ be a function that is continuous and differentiable on the interval $(2, 7)$.

Particular values of $f$ and $f'$ are given in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-1</td>
<td>3</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>4</td>
<td>-3</td>
<td>-2</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Find $L(x)$, the linear approximation to $f$ at $a = 4$.

$L(x) =$

(b) Use the linearization from part (a) to estimate $f(4.2)$.

$f(4.2) \approx$

(c) Find the **average rate of change**, $AR$, of the function $f$ on the interval $[4, 6]$. Show work.

$AR =$

(d) Circle the interval for which the following statement is true.

The Mean Value Theorem guarantees that there exists a number $c$ in this interval such that $f'(c) = 2$.

i. $(3, 5)$

ii. $(4, 6)$

iii. $(3, 4)$

iv. No previous answer is correct.
5. (14+3 pts) Show your work!

(a) (14 pts) First, state the **form** of the limit and indicate whether the form is **indeterminate or not** by circling Yes or No. Then, evaluate the limit or say that the limit “does not exist”. You may use L'Hôpital's Rule.

i. (10 pts) \( \lim_{x \to 0^+} 6x^2 \ln(x) \)

\[ \text{FORM=} \quad \text{Is the form indeterminate? Circle: Yes or No} \]

\[ \lim_{x \to 0^+} 6x^2 \ln(x) = \]

ii. (4 pts) \( \lim_{x \to 1} \frac{6 \ln(x)}{x^2} \)

\[ \text{FORM=} \quad \text{Is the form indeterminate? Circle: Yes or No} \]

\[ \lim_{x \to 1} \frac{6 \ln(x)}{x^2} = \]

(b) (BONUS 3 pts) Compute the limit. (Hint: \( \sum_{k=1}^{n} C = n \cdot C \))

\[ \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} = \]
6. (18 pts) A right triangle is constructed in the first quadrant. Its hypotenuse runs from the origin to a point on the line \( y = 2 - x \). One of its sides lies on the \( y \)-axis, and the other side is parallel to the \( x \)-axis (see figure). Find the values \( x \) and \( y \) which \textbf{minimize the perimeter} of the triangle.

Solve the problem by performing the following steps.

(a) Label the figure above.

(b) Express \( P \), the perimeter of the triangle, as a function of \( x \). What is the domain of the function \( P \)?

\[
P(x) = \quad \text{Domain of } P =
\]

(c) Solve the problem. Show your work and justify your answer!