Name: ______________________

OSU name.#: ______________________

Lecturer: ______________________

Recitation Instructor: ______________________

Recitation Time: ______________________

Instructions.

- **Show all relevant work** to receive full credit on Problems 1, 2, 3, 4(a)-(c), 5, and 6.
  Incorrect answers with substantially correct work may receive partial credit.
  Unsupported answers may receive no credit.
- Problem 4(d) is **multiple choice**. Circle *exactly one* choice.
  Ambiguous markings may receive no credit.
- Give **exact** answers unless instructed to do otherwise.
- **No calculators, phones, or other devices may be used** during the exam.
  Do not have these devices out!
- No notes or references are permitted.
- The allotted time for this exam is **55 minutes**.
- The exam consists of 6 problems starting on Page 2 and ending on Page 7. Page 8 is blank.
  Check that your exam is complete before you begin.

Problem 1 [16 points]	
Problem 2 [18 points]	
Problem 3 [16 points]	
Problem 4 [18 points]	
Problem 5 [14 +3 points]	
Problem 6 [18 points]	
Total [103 points]	

1. (16 pts) Sketch the graph of a function f satisfying **all** of the following conditions:

   (a) f is **even**;

   (b) f(0)=5; f is continuous, but **not differentiable** at $x = 0$;

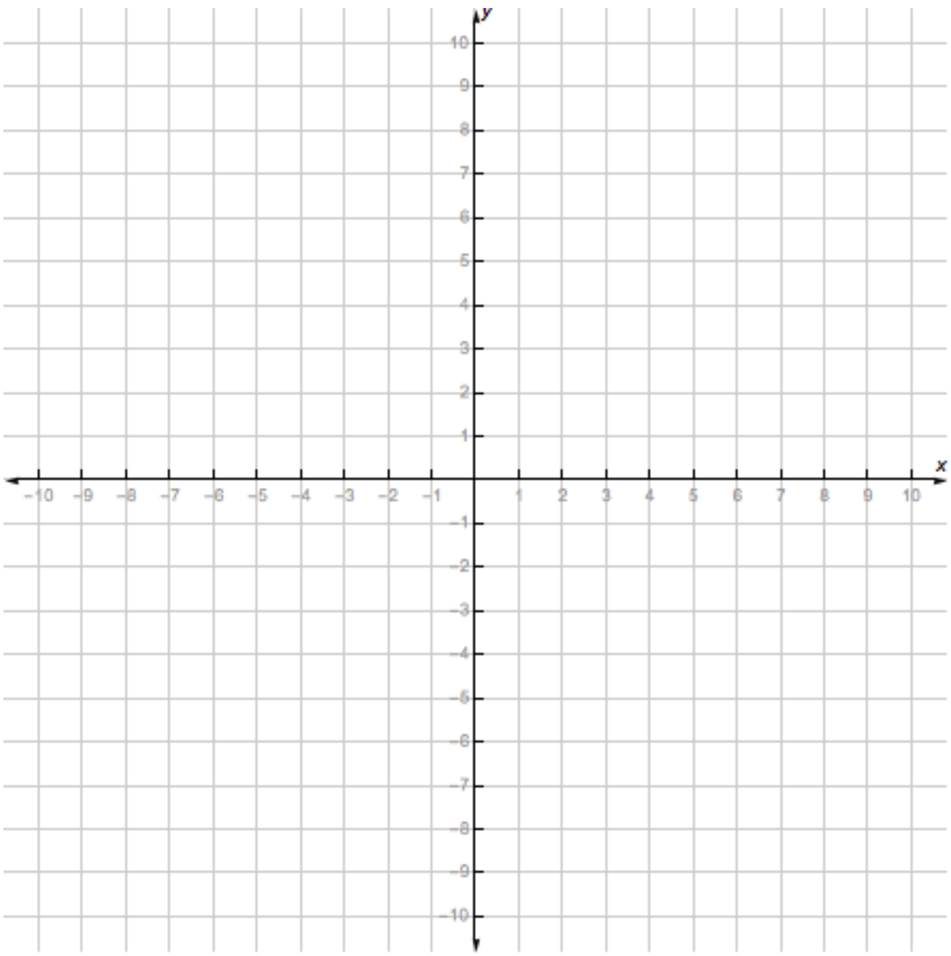
   (c) $\lim_{x\to+\infty} f(x) = 5$, $\lim_{x\to 6} f(x) = -\infty$;

   (d) $f'(x) > 0$ on $(6, +\infty)$;

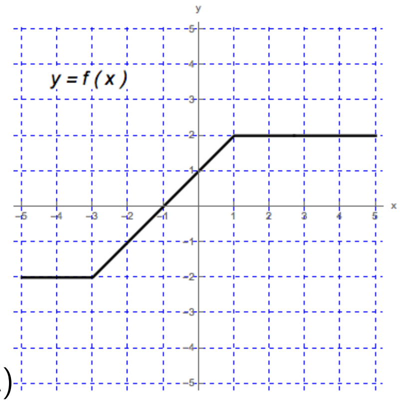
   (e) $f'(x) < 0$ on $(0, 6)$;

   (f) $f''(x) > 0$ on $(0, 3)$;

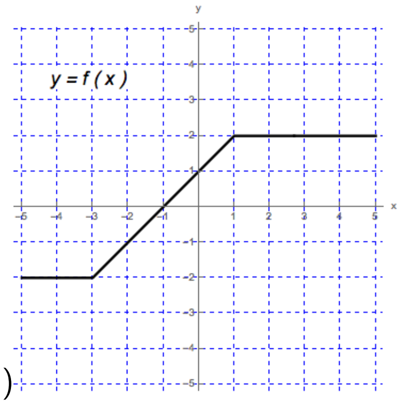
   (g) $f''(x) < 0$ on $(3, 6)$ and $(6, +\infty)$.



2. (18 pts) Show your work!
The function f is continuous and piecewise linear on $[-5, 5]$. The graph of f is shown twice below.



(A)



(B)

(a) Compute the **right** Riemann sum of f on $[-5, 5]$ for $n = 2$.
Illustrate this Riemann sum by sketching appropriate rectangles in figure (A) above.

(b) Express the limit of Riemann sums of the function f on the interval $[-5, 5]$ as a definite integral and use geometry to evaluate it. (You may use figure (B) for this computation.)

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

(c) Use geometry and properties of definite integrals to evaluate the integral.
(You may use figure (B) for this computation).

$$\int_{-5}^{5} (2f(x) - 1)\, dx$$

3. (16 pts) Consider an object moving along a horizontal line.
   The velocity function v and the initial position $s(0)$ of the object are given.

$$\begin{cases} v(t) &= t - \frac{\pi}{4}\sin\left(\frac{\pi}{4}t\right), \qquad 0 \leq t \leq 8; \\ s(0) &= 0 \end{cases}$$

   (a) Find $s(t)$, the position of the object at the time t, for $0 \leq t \leq 8$.

   (b) Find the **displacement** of the object over the time interval $[0, 4]$.

   (c) Express the **displacement** of the object over the time interval $[0, 4]$ as a definite integral.

4. (18 pts) Let f be a function that is continuous and differentiable on the interval $(2, 7)$.

   Particular values of f and f' are given in the table below.

x	3	4	5	6
$f(x)$	-1	3	6	7
$f'(x)$	4	-3	-2	5

   (a) Find $L(x)$, the linear approximation to f at $a = 4$.

   $L(x) =$

   (b) Use the linearization from part (a) to estimate $f(4.2)$.

   $f(4.2) \approx$

   (c) Find the **average rate of change**, AR, of the function f on the interval $[4, 6]$. Show work.

   $AR =$

   (d) Circle the interval for which the following statement is true.

   The Mean Value Theorem guarantees that there exists a number c in this interval such that $f'(c) = 2$.

   i. $(3, 5)$

   ii. $(4, 6)$

   iii. $(3, 4)$

   iv. No previous answer is correct.

5. (14+3 pts) Show your work!

   (a) (14 pts) First, state the **form** of the limit and indicate whether the form is **indeterminate or not** by circling Yes or No. Then, **evaluate the limit** or say that the limit "does not exist".
   You may use L'Hôpital's Rule.

      i. (10 pts) $\lim_{x \to 0^+} 6x^2 \ln(x)$

      FORM= Is the form indeterminate? Circle: Yes or No

      $\lim_{x \to 0^+} 6x^2 \ln(x) =$

      ii. (4 pts) $\lim_{x \to 1} \frac{6 \ln(x)}{x^2}$

      FORM= Is the form indeterminate? Circle: Yes or No

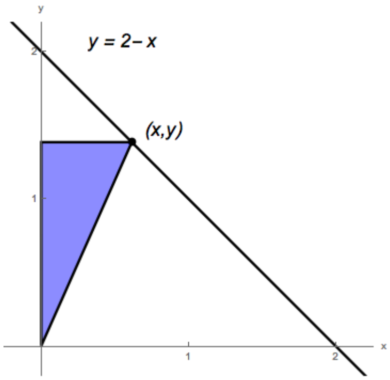
      $\lim_{x \to 1} \frac{6 \ln(x)}{x^2} =$

   (b) (BONUS 3 pts) Compute the limit. (Hint: $\sum_{k=1}^{n} C = n \cdot C$)

$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n}$$

6. (18 pts) A **right** triangle is constructed in the first quadrant.

   Its hypotenuse runs from the origin to a point on the line $y = 2 - x$.
   One of its sides lies on the $y-$axis, and the other side is parallel to the $x-$axis (see figure).
   Find the values x and y which **minimize the perimeter** of the triangle.



   Solve the problem by performing the following steps.

   (a) Label the figure above.

   (b) Express P, the perimeter of the triangle, as a function of x. What is the domain of the function P?

   $P(x) =$ Domain of P=

   (c) Solve the problem. Show your work and justify your answer!

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