PV Numbers and Salem Numbers

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I. Algebraic numbers
   A. Definition   B Examples   C How many are there? (countable)

II. Minimal polynomials
   A. Definition. Among all the monic polynomials in **Q**[x] having θ as root, one will divide all the rest
      1. Algebraic integers: Minimal polynomial has integer coefficients
      2. Examples (rational algebraic integers are actual integers)
   B. Galois conjugates

III. PV numbers – motivation and definition
   A. Almost integers (irrational numbers that are surprisingly close to integers)
   B. Numbers whose powers are almost integers (golden mean)
   C. PV numbers first studied by Axel Thue and G. H. Hardy, and Hardy's Indian student Vijayaraghavan
      1. Irrational real algebraic integers
      2. Greater than 1
      3. Galois conjugates ALL less than 1 in absolute value
      4. Diophantine approx: PV numbers have high powers = almost integers

IV. Related phenomenon: Salem numbers
   A. Irrational real algebraic integers
   B. Greater than 1
   C. All Galois conjugates less than or equal to 1 in absolute value
   D. At least one conjugate equal in absolute value to 1

V. Small PV numbers
   A. A note on size: every PV number is bigger in absolute value than the constant (integer) coefficient of its minimal polynomial
   B. What is the smallest limit point of the PV numbers? (Answer: Golden mean)
   C. PV numbers form a closed set, bounded below: What the smallest PV number?
   D. Examples of small PV numbers: Quadratic irrationals

VI. Diophantine characterization of PV numbers
   A. Square-summability condition characterizes PV numbers among all real numbers
   B. Limit-goes-to-0 condition characterizes PV numbers among all real algebraic numbers (open question: among all real numbers?)
   C. A converse also holds

VII. Facts and conjectures involving Salem numbers
   A. Fact: Every trace is realized among Salem numbers
   B. Conjecture: The Salem numbers also form a closed set