Ph.D. Algebra Qualifying Exam
6112

August, 2015
Proctor: Jim Cogdell

Directions

1. Initial the roster sheet and enter a code name for yourself that is different from any code name that has already been entered.

2. Answer each question on a separate sheet or sheets of paper, and write your *code name* and the *problem number* on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.

3. Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment you use in this respect is an important part of the exam.

4. This is a closed book, closed notes exam.

1. Let $\mathcal{A}$ be an abelian category.
   (a) Defines what it means for a morphism to be a monomorphism and to be an epimorphism.
   (b) Show that a morphism is a monomorphism if and only if its kernel is 0.
2. Prove that an abelian group T is divisible iff T is injective as a $\mathbb{Z}$-module.
3. Let R be a commutative PID. Let $a \in R$, $a \neq 0$, and let $M = R/(a)$. For any R module N show that
$$Ext^1(M, N) \cong N/aN.$$
4. Let $F_r = \mathbb{F}_{p^r}$ be the finite field of p^r elements and let $N_{F_1}^{F_r}$ denote the norm homomorphism from F_r^* to F_1^*. Calculate $|ker(N_{F_1}^{F_r})|$, the cardinality of the kernel of the norm map.
5. Let K be a finite extension of $\mathbb{Q}$ containing $\zeta = e^{2\pi i/n}$. If $K(\sqrt[n]{\alpha})$ and $K(\sqrt[n]{\beta})$ are two Kummer extensions of K of degree n such that $K(\sqrt[n]{\alpha}) = K(\sqrt[n]{\beta})$, show that $\alpha = \beta^r c^n$ for some integer r with $(r, n) = 1$ and some $c \in K$.