A cuspidal analogue of Titchmarsh's divisor problem

Abstract

Given the Fourier coefficients $a(n)$ of a holomorphic cusp form for the modular group, we will show that

$$\sum_{\substack{p \le X \\ p \text{ prime}}} a(p-1) \ll X^{391/392+\varepsilon}$$

for any $\varepsilon > 0$ for large X. Similarly

$$\sum_{n \le X} \mu(n) a(n-1) \ll X^{391/392+\varepsilon}.$$

We will sketch the proofs, which require establishing non-trivial bounds for sums of Kloosterman sums and shifted convolutions of the coefficients which are better in the ranges required than known estimates. These are then used to bound bilinear forms in $a(mn-1)$, which in conjunction with previous work of the speaker proves the main results.