What is Quantum Computing?

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June 28, 2012

Definitions

Qubit The unit of information in quantum computing. Defined as a superposition of two states $|0\rangle$ and $|1\rangle$, and represented as $|\psi\rangle = a|0\rangle + b|1\rangle$, $a, b \in \mathbb{C}$, where $|a|^2 + |b|^2 = 1$. State can also be represented as the matrix $\begin{bmatrix} a \\ b \end{bmatrix}$. Physically, $|a|^2$ and $|b|^2$ represent the probability that, upon measurement, 0 or 1 will be obtained, respectively.

N-qubit system A superposition of 2^n states, represented as $|\psi\rangle = \sum_{i=0}^{2^n-1} c_i |x_i\rangle$, $c_i \in \mathbb{C}$, where $\sum_{i=0}^{2^n-1} c_i = 1$ and x_i is the i-th bitstring (0...0, 0...1, ..., 1...1) of size n. Conveniently represented as $\begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix}$.

Quantum gate Mathematically, a $2^n \times 2^n$ linear map U from one n-bit quantum state to the next. Produces a new quantum state $|\psi'\rangle = U|\psi\rangle = U \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} c'_0 \\ c'_1 \\ \vdots \\ c'_n \end{bmatrix}$. U must ensure that $\sum_{i=0}^{2^n-1} c'_i = 1$, i.e. U must be a unitary transformation.

Hadamard gate One of the universal quantum operations, defined by $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Important because it transforms any pure state into a superposition of its possible states.

Phase gate Another universal quantum operation, defined by $\Phi_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$.

Controlled-NOT (CNOT) gate A universal quantum operation on two qubits simultaneously, defined by

$$U_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Bit-flip gate Defined by $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Flips the coefficients of $|0\rangle$ and $|1\rangle$.

Phase-flip gate Defined by $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Flips $b|1\rangle$ to $-b|1\rangle$.

References

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