

WHAT IS... THE HOT SPOT THEOREM?

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ABSTRACT. Given a real number $x \in [0, 1)$ with base- b expansion $x = 0.a_1a_2a_3\dots$, one can ask many questions about how well distributed the digits are. The *crème de la crème* of such distributions is normality, where every string of digits appears with the same (limiting) frequency as every other string of the same length. It can be shown that almost all real numbers have normal expansions.

One can then ask how close one can get to normality without being normal. In this talk we will introduce the weak hot spot theorem (also known as the Pyatetskii-Shapiro normality criterion), which says that if a number is not normal, then the distribution of its digits must be very far from normal. This is an amazing tool because it weakens the conditions needed to prove normality from exact limits to simple upper bounds. We can use this to easily prove facts about normal numbers, such as Wall's Theorem, which says that if we select along an arithmetic progression of the digits of a normal number (such as $0.a_1a_4a_7a_{10}\dots$), then we get another normal number.