What is Lüroth Expansion?

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Abstract

While representing real numbers as a (possibly) infinite sequence of digits through base b expansion or through continued fractions already generates quite a bit of theory, there still do exist many other interesting, albeit lesser known, methods of doing so. One of these, called *Lüroth expansion*, takes a real number $r \in (0, 1]$ and writes it

$$r = \frac{1}{a_1} + \frac{1}{a_1(a_1-1)a_2} + \cdots + \frac{1}{a_1(a_1-1)\cdots a_{n-1}(a_{n-1}-1)a_n} + \cdots,$$

where $a_i \geq 2$ for each i. Each such r can be uniquely expressed in this way, and it is not hard to see that, for any sequence $\{a_n\}_{n=1}^{\infty}$ of integers $a_n \geq 2$, the above sum converges.

While in base b we have an easily defined shift operator $T_b x = bx - \lfloor bx \rfloor$ that forgets about the first digit of x, for the Lüroth expansion we have the *Lüroth shift*

$$Tx := \lfloor \frac{1}{x} \rfloor (\lfloor \frac{1}{x} \rfloor + 1)x - \lfloor \frac{1}{x} \rfloor.$$

In this talk I investigate the properties of T and some of their stochastic and metric usefulness in studying Lüroth expansions.