What is a Weierstrass Point?

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Abstract

Given a graph, tropical curve, or Riemann surface, we may introduce the so-called chip-firing game on it. In a chip-firing game, we are given a starting configuration of chips in which finitely many points on our structure are given a non-zero (possibly negative) number of chips. Our goal is to, through a series of valid "chip firing moves," redistribute the wealth of our structure so that no point is in debt, having a negative number of chips. We may restrict our attention to starting configurations which consist of a single point with many chips and several other points in debt. We may determine how proficient a given point with a given number of chips is at removing a certain amount of debt from the rest of the graph, curve, or surface. As it turns out, some points, called Weierstrass points, are better at it than others, needing fewer chips to cancel a given amount of debt.