

# Moonshine

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## Abstract

Moonshine is the surprising connection between two very different mathematical objects, the monster group and the  $j$ -function. In the classification of finite simple groups, the monster group is the largest sporadic group. The  $j$ -function is a modular form that is an invariant of isomorphism classes of elliptic curves. John McKay and John Thompson noticed a relationship between dimensions of irreducible representations of the monster group and Fourier coefficients of the  $j$ -function. This seemingly coincidental relationship led McKay and Thompson to conjecture the existence of an infinite dimensional representation of the monster group reflecting these relationships. This representation was shown to exist, and many other interesting facts arose. In this talk I will describe the history of this phenomenon and describe further interesting facts that have fallen out of this relationship as it's been studied.

## References

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