What is the Toeplitz Conjecture?

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Abstract

The Toeplitz conjecture (also known as the inscribed square problem or the square peg problem) is an open question which asks: Does every Jordan curve admit an inscribed square? That is, does every simple closed curve in the plane contain four vertices of a square? Otto Toeplitz posed the question in 1911, but despite the problem's longevity and simple statement, it remains unsolved. In this talk we discuss some results that came out of consideration of this problem. In particular, we present some stronger conditions under which the problem was solved, and we also see some variations of the problem. At the end we give a nice proof of one of these variations.

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