I. Parametric Equations

a) \( x = 2t^2, \quad y = 3t \).

Solve for \( t \) from \( y \) since the expression is easier:

\[
\frac{1}{3} y
\]

So:

\[
\begin{align*}
x &= 2 \left( \frac{1}{3} y \right)^2 \\
x &= \frac{2}{9} y^2
\end{align*}
\]

As \( t \) increases, \( y \) increases, so the positive orientation is as indicated.

b) \( x = 4 \sin t + 1, \quad y = 4 \cos t \)

\[
\begin{align*}
x - 1 &= 4 \sin t \\
(\sqrt{x - 1})^2 &= 16 \sin^2 t
\end{align*}
\]

So:

\[
(\sqrt{x - 1})^2 + y^2 = 16 \sin^2 t + 16 \cos^2 t
\]

\[
(\sqrt{x - 1})^2 + y^2 = 16
\]

At \( t = 0 \), start at \((1, 4)\). As \( t \) increases, \( x \) increases, so the orientation is as shown.

c) \( x = 4e^t, \quad y = e^{3t} \)

\[
e^t = \frac{1}{4} x
\]

Since \( y = e^{3t} = (e^t)^3 \)

\[
\gamma = \left( \frac{1}{4} x \right)^3
\]

\[
\gamma = \frac{1}{64} x^3
\]

Note: \( x, y \) are always positive! As \( t \to -\infty \), \((x, y) \to (0, 0)\).
d) \( x = 2\cos t, \quad y = \sin t \)
\[
\frac{1}{2} x = \cos t \quad y = \sin t
\]
\[
\frac{1}{4} x^2 = \cos^2 t \quad y^2 = \sin^2 t.
\]
\[
\frac{1}{4} x^2 + y^2 = \cos^2 t + \sin^2 t
\]
\[
\frac{x^2}{4} + y^2 = 1 \quad \text{ Ellipse}
\]

2. There are many different answers to these. Here are a few:

a) We can take \( x = 2 \cos t, \quad y = 1 = 2 \sin t \).

But, when \( t = 0 \): \( x = 2, \quad y = 1 = 0 \).

So, try: \( x = 2 \sin t, \quad y = 1 + 2 \cos t \).

A quick check shows \( x(0) = 0, \quad y(0) = 3 \).

b) The decumanus above is traced out clockwise. Another would be

\( x = -2 \cos t, \quad y = 1 + 2 \sin t \).

Note: The \(-\) changes the direction.

c) \( x = 2 \cos t, \quad y = 1 + 2 \sin t \) will trace out the circle.

It starts at \((2,0)\). We need 2 full rotations to occur between \( t = 0 \) and \( t = 1 \), so we need the argument of \( \cos \) to be \( 0 \) at \( t = 0 \) and \( 4\pi \) when \( t = 1 \):

\[ x = 2 \cos 4\pi t, \quad y = 1 + 2 \sin 4\pi t \]

3a) \( x = at \rightarrow t = \frac{x}{a} \).

Since \( y = bt \), \( y = b(\frac{x}{a}) \) or \( y = \frac{b}{a} x \).
b) When \( t = 0 \), \( y = b \), so try \( y = bt + b \).

We need \( y = \frac{b}{a} x \)

\[ bt + b = \frac{b}{a} x. \]

\[ b(t + 1) = \frac{b}{a} x \]

\[ a(t + 1) = x \]

Note that we can simply replace \( t \to t+1 \) in the original expression!

4 a) \[ \frac{dy}{dx} = \frac{dy/\,dt}{dx/\,dt} = \frac{2t}{3t^3 - 3} \]

b) **Horizontal Tangents**: occur when the numerator is 0:

\[ 2t = 0 \Rightarrow t = 0 \]

When \( t = 0 \):

\[ y = 0^2 = 0, \quad x = 0^3 - 3(0) \]

So the horizontal tangent is \( y = 0 \) and occurs at \((0, 0)\)

**Vertical Tangents**: occur when the denominator is 0:

\[ 3t^3 - 3 = 0 \]

\[ t = 1, -1 \]

When \( t = 1 \):

\[ y = (1)^2 = 1 \]

\[ x = 1^3 - 3(1) = -2 \]

So the vertical tangent is \( x = -2 \) and occurs at \((-2, 1)\)

When \( t = -1 \):

\[ y = (-1)^2 = 1 \]

\[ x = (-1)^3 - 3(-1) = -2 \]

So the vertical tangent is \( x = 2 \) and occurs at \((2, 1)\)
c) When $t=2$:
\[ \frac{dy}{dx} = \frac{2(2)}{3(2)^2 - 3} = \frac{4}{9}. \]
- $x = (2)^3 - 3(2) = 2$
- $y = (2)^2 = 4$

So:
\[ y - y_0 = m(x - x_0) \quad \text{equation of a line} \]
\[ y - 4 = \frac{4}{9} (x - 2) \]

d) When $(x, y) = (2, 1)$, we need to find $t$.
\[ x = 2 = t^3 - 3t \quad \text{Hard to solve!} \]

- Try $y$ instead:
\[ y = 1 = t^2 \Rightarrow t = 1 \text{ or } -1. \]
- Need to find which makes $x = 2$:
  \[ t = 1: \quad x = 1^3 - 3(1) = -2 \quad \times \]
  \[ t = -1: \quad x = (-1)^3 - 3(-1) = 2 \quad \checkmark \]

So: $t = -1$!
This corresponds to the solution found in 2b):

The tangent line there was found to be $x = 2$

5. a) \[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t-1}{2t-6} \]

b) The horizontal tangents:
Set $t - 1 = 0$
\[ t = 1 \]

When $t = 1$:
\[ x = 1^2 - 6(1) + 1 = -4 \]
\[ y = \frac{1}{2} (1)^2 - 1 = -\frac{1}{2} \]
\[ \Rightarrow \text{The horizontal tangent is } y = -\frac{1}{2} \text{ and occurs at } (-4, -\frac{1}{2}) \]
c). This was just found in b).

d) When \((x, y) = (1, 12)\), we need to find \(t\).

\[
x = 1 = t^2 - 6t + 1
\]

\[
t^2 - 6t = 0
\]

\[
t(t - 6) = 0 \Rightarrow t = 0, t = 6.
\]

We need to find which \(t\)-value gives \(y = 12\):

\[
t = 0: \quad y = \frac{1}{2} (0)^2 - 0 = 0 \quad \times
\]

\[
t = 6: \quad y = \frac{1}{2} (6)^2 - 6 = 12 \quad \checkmark
\]

So:

\[
\frac{dy}{dx} \bigg|_{t=6} = \frac{6 - 1}{2(6) - 6} = \frac{5}{6} \quad \leftarrow \text{slope}
\]

Tang line:

\[
y - y_0 = m(x - x_0)
\]

\[
y - 12 = \frac{5}{6} (x - 1)
\]

6. Consider this visually:

\[
(x, y) = (0, 1)
\]

Way 1: \(r = 1, \ \theta = \frac{\pi}{2}\) (rotate \(r = 1\) CCW by \(\frac{\pi}{2}\)).

Way 2: \(r = 1, \ \theta = -\frac{3\pi}{2}\) (rotate \(r = 1\) CW by \(-\frac{3\pi}{2}\)).

Way 3: \(r = -1, \ \theta = \frac{3\pi}{2}\) (rotate \(r = -1\) CCW by \(\frac{3\pi}{2}\)).

Way 4: \(r = -1, \ \theta = -\frac{\pi}{2}\) (rotate \(r = -1\), CW by \(-\frac{\pi}{2}\)).

* \(\theta > 0 \leftrightarrow \text{CCW rotation}\)

* \(\theta < 0 \leftrightarrow \text{CW rotation}\)
7. First note \( r^2 = x^2 + y^2 \Rightarrow r^2 = (-\sqrt{3})^2 + (\sqrt{3})^2 \)
\[ r^2 = 2 + 2 \]
\[ r = \pm 2. \]
Once again, think visually:
\[ (-\sqrt{3}, -\sqrt{3}) \]
\[ (\sqrt{3}, -\sqrt{3}) \]
Way 1: \( r = 2, \, \theta = \frac{3\pi}{4} \)
Way 2: \( r = 2, \, \theta = \frac{3\pi}{4} \)
Way 3: \( r = 2, \, \theta = \frac{\pi}{4} \)
Way 4: \( r = 2, \, \theta = -\frac{\pi}{4} \)

8. \( x = r \cos \theta = 2 \cos \frac{\pi}{3} = 2 \left( \frac{1}{2} \right) = 1 \)
\( y = r \sin \theta = 2 \sin \frac{\pi}{3} = 2 \left( \frac{\sqrt{3}}{2} \right) = 2\sqrt{3} \)
So \( (x, y) = (2, 2\sqrt{3}) \)

9.a) \( r = 4 \text{ sec } \theta \) ← convert to \( \sin \theta, \cos \theta \)
\( r = \frac{4}{\cos \theta} \)
\( r \cos \theta = 4 \)
\[ x = 4 \]

b) \( r^2 = \tan \theta \) ← convert to \( \sin \theta, \cos \theta \)
\( r^2 = \frac{\sin \theta}{\cos \theta} \) ← make \( r \sin \theta, \, r \cos \theta \) appear
\( r^2 = \frac{r \sin \theta}{r \cos \theta} \)
\( x^2 + y^2 = \frac{y}{x} \)
c) \[ r^2 = \sin \theta \cos \theta \]

\[ r^2 = r^2 \sin \theta \cos \theta \]

\[ r^2 = (r \sin \theta)(r \cos \theta) \]

\[ x^2 + y^2 = r^2 \]

\[ x^2 + y^2 = 25 \]

d) \[ r = 5 \]

\[ \sqrt{x^2 + y^2} = 5 \]

\[ x^2 + y^2 = 25 \]

10. \[ r = 2 \cos \theta \]

a) \[ \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \]

Note: \[ \chi = r \cos \theta = (2 \cos \theta) \cos \theta = 2 \cos^2 \theta \]

\[ \chi = 4 \cos \theta \sin \theta \]

\[ \gamma = r \sin \theta = (2 \cos \theta) \sin \theta \]

\[ \frac{dy}{d\theta} = 2 \cos^2 \theta - 2 \sin^2 \theta \]

So: \[ \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2 \cos^2 \theta - 2 \sin^2 \theta}{4 \cos \theta \sin \theta} \]

b) Horizontal Tangents: \[ 2 \cos^2 \theta - 2 \sin^2 \theta = 0 \]

\[ 2 \cos^2 \theta - 2 \sin^2 \theta = 2 \cos^2 \theta - 2 \sin^2 \theta = 0 \]

\[ 2 \cos^2 \theta - 2 \sin^2 \theta = 4 \sin \theta \]

\[ \pm \sqrt{\frac{1}{2}} = \sin \theta \]

Noting \[ \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{2} \]

We need the \( x, y \)-values.
<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$r = 2 \cos \theta$</th>
<th>$x = r \cos \theta$</th>
<th>$y = r \sin \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/4$</td>
<td>$\sqrt{2}$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$3\pi/4$</td>
<td>$-\sqrt{2}$</td>
<td>$1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$5\pi/4$</td>
<td>$-\sqrt{2}$</td>
<td>$1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$7\pi/4$</td>
<td>$\sqrt{2}$</td>
<td>$1$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

The horizontal tangents are:

- $y = 0$ at $(1, 1)$
- $y = -1$ at $(1, -1)$

**Vertical tangents:** Need $\frac{dy}{dx}$ is undefined

$4 \cos \theta \sin \theta = 0$  
$\Rightarrow \sin \theta = 0$ or $\cos \theta = 0$  
$\Rightarrow \theta = 0, \pi$ or $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$r = 2 \cos \theta$</th>
<th>$x = r \cos \theta$</th>
<th>$y = r \sin \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$2$</td>
<td>$2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$-2$</td>
<td>$2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$3\pi/2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

The vertical tangents are:

- $x = 2$ at $(2, 0)$
- $x = 0$ at $(0, 0)$

**Note:** Graphically, $r = \cos \theta$ is a circle of radius $\frac{1}{2}$ centered at $(1, 0)$.

$r = 2 \cos \theta$
$r^2 = 2r \cos \theta$
$x^2 + y^2 = 2x$
$x^2 - 2x + 1 + y^2 = 1$
$(x-1)^2 + y^2 = 1$
c) \[ \frac{dy}{dx} \bigg|_{\theta = \frac{\pi}{6}} = \frac{2 \cos^2 \frac{\pi}{6} - 2 \sin^2 \frac{\pi}{6}}{4 \cos \frac{\pi}{6} \sin \frac{\pi}{6}} \]

\[ = \frac{2 \left( \frac{\sqrt{3}}{2} \right)^2 - 2 \left( \frac{1}{2} \right)^2}{4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}} \]

\[ = 2 \cdot \frac{3}{4} - 2 \left( \frac{1}{4} \right) \]

\[ \frac{dy}{dx} \bigg|_{\theta = \frac{\pi}{6}} = \frac{1}{\sqrt{3}} \]

- Find \( r \):

\[ r = 2 \cos \theta = 2 \cos \frac{\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} \Rightarrow r = \sqrt{3} \]

- Find \( x, y \):

\[ x = r \cos \theta = \sqrt{3} \cos \frac{\pi}{6} = \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2} \]

\[ y = r \sin \theta = \sqrt{3} \sin \frac{\pi}{6} = \sqrt{3} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2} \]

- Find the line:

\[ y - y_0 = m(x - x_0) \]

\[ y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}} \left( x - \frac{3}{2} \right) \]

10. \( r = 2 + 2 \sin \theta \).

- \( x = r \cos \theta = (2 + 2 \sin \theta) \cos \theta = 2 \cos \theta + 2 \sin \theta \cos \theta \).

\[ \frac{dx}{d \theta} = -2 \sin \theta + 2 \cos^2 \theta - 2 \sin^2 \theta. \]

- \( y = r \sin \theta = (2 + 2 \sin \theta) \sin \theta = 2 \sin \theta + 2 \sin^2 \theta \)

\[ \frac{dy}{d \theta} = 2 \cos \theta + 4 \sin \theta \cos \theta. \]

\[ \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d \theta}}{\frac{dx}{d \theta}} = \frac{2 \cos \theta + 4 \sin \theta \cos \theta}{-2 \sin \theta + 2 \cos^2 \theta - 2 \sin^2 \theta} \]

Noting \( \cos^2 \theta = 1 - \sin^2 \theta \), we can write after some algebra:

\[ \frac{dy}{dx} = -\frac{\cos \theta (1 + 2 \sin \theta)}{2 \sin^2 \theta + \sin \theta - 1} = -\frac{\cos \theta (1 + 2 \sin \theta)}{(2 \sin \theta - 1)(1 + \sin \theta)}. \]
b) **Horizontal Tangents:** \( \frac{dy}{dx} = 0 \Rightarrow \cos \theta = 0 \), or \( 1 + 2 \sin \theta = 0 \)
\[ \begin{align*}
\theta &= \frac{\pi}{2}, \frac{3\pi}{2} \\
\sin \theta &= \frac{-1}{2} \\
\theta &= \frac{7\pi}{6}, \frac{11\pi}{6}
\end{align*} \]

Note when \( \theta = \frac{3\pi}{2} \), the denominator is 0 as well!

If \( \lim_{\theta \to \frac{3\pi}{2}} \frac{\cos \theta}{1 + \sin \theta} \) exists, the limit laws tell us

\[
\lim_{\theta \to \frac{3\pi}{2}} \frac{\cos \theta}{1 + \sin \theta} = \lim_{\theta \to \frac{3\pi}{2}} \frac{1 + 2 \sin \theta}{1 - 2 \sin \theta} = \lim_{\theta \to \frac{3\pi}{2}} \frac{\cos \theta}{1 + \sin \theta} \lim_{\theta \to \frac{3\pi}{2}} \frac{1 + 2 \sin \theta}{1 - 2 \sin \theta}.
\]

Note \( \lim_{\theta \to \frac{3\pi}{2}} \frac{\cos \theta}{1 + \sin \theta} = \frac{-1}{0} = -\frac{(-1)}{0} \) \( \Rightarrow \) DNE!

Thus, \( \lim_{\theta \to \frac{3\pi}{2}} \frac{dy}{dx} \) DNE and this corresponds to a vertical tangent!

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( r = 2 + 2 \sin \theta )</th>
<th>( x = r \cos \theta )</th>
<th>( y = r \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>( \frac{7\pi}{6} )</td>
<td>1</td>
<td>(-\frac{\sqrt{3}}{2})</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>( \frac{11\pi}{6} )</td>
<td>1</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
</tbody>
</table>

**Horizontal Tangents:**

\( y = 4 \) at \((x, y) = (0, 4)\)

\( y = -\frac{1}{2} \) at \((x, y) = (-\frac{\sqrt{3}}{2}, -\frac{1}{2})\)

\( y = \frac{1}{2} \) at \((x, y) = (\frac{\sqrt{3}}{2}, \frac{1}{2})\)
Vertical Tangents: \( \frac{dy}{dx} \) is undefined \( \Rightarrow \) \( \pm \sin \theta = 0 \) or \( 1 - 2 \sin \theta = 0 \).

\[ \theta = \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{3\pi}{6}, \frac{5\pi}{6}. \]

When \( \theta = \frac{3\pi}{2} \), the numerator is also 0, but we showed previously the limit of \( \frac{dy}{dx} \) as \( \theta \to \frac{3\pi}{2} \) DNE, so \( \theta = \frac{3\pi}{2} \) \( \Rightarrow \) a vertical tangent.

\[
\begin{array}{|c|c|c|c|}
\hline
\theta & r = 2 + 2 \sin \theta & x = r \cos \theta & y = r \sin \theta \\
\hline
\frac{3\pi}{2} & 0 & 0 & 0 \\
\frac{\pi}{6} & 3 & \frac{3\sqrt{3}}{2} & \frac{3}{2} \\
\frac{5\pi}{6} & -3 & -\frac{3\sqrt{3}}{2} & \frac{3}{2} \\
\hline
\end{array}
\]

Vertical Tangents:

\[
\begin{align*}
x &= 0 & \text{at} & (0, 0) \\
x &= \frac{3\sqrt{3}}{2} & \text{at} & (\frac{3\sqrt{3}}{2}, \frac{3}{2}) \\
x &= -\frac{3\sqrt{3}}{2} & \text{at} & (-\frac{3\sqrt{3}}{2}, \frac{3}{2})
\end{align*}
\]

Note: The graph of \( r = 2 + 2 \sin \theta \) is shown below:

![Graph of r = 2 + 2 sin(θ)](image)

The positions of the horizontal and vertical tangents are again justified graphically!
12.

- We have 2 curves
- If we draw a ray extending from the origin into the region, the curve it hits changes! \( \Rightarrow \) mult integrals
- From symmetry, the area of the shaded region is twice the area of the dark region.

\[
\text{Int R}_A: \quad \sqrt{1} = \sqrt{\cos \theta} \\
\frac{1}{2} = \cos \theta \\
\theta = \frac{\pi}{3}, \frac{2\pi}{3}
\]

\[
A_I = \frac{1}{2} \int_0^{\pi/3} \frac{1}{2} \sin^2 \theta \, d\theta = \frac{1}{2} \int_0^{\pi/3} (\sqrt{3})^2 \, d\theta = \frac{\pi}{12}
\]

\[
A_{II} = \frac{1}{2} \int_{\pi/3}^{\pi/2} \frac{1}{2} \sin^2 \theta \, d\theta = \frac{1}{2} \int_{\pi/3}^{\pi/2} \cos \theta \, d\theta = \frac{1}{2} \sin \theta \bigg|_{\pi/3}^{\pi/2} = \frac{1}{2} - \frac{\sqrt{3}}{2}
\]

\[
A = 2(A_I + A_{II}) = 2 \left[ \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{2} \right]
\]

\[
A = \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}
\]

13.

- We have 2 curves
- From symmetry, the total area is 8 times the area of the darkly shaded region
- In this region there is an inner and outer curve!
Limits: $\Theta = 0$ (by inspection)

$$4 \cos 2\Theta = 2$$

$$\cos 2\Theta = \frac{1}{2} \Rightarrow 2\Theta = \frac{\pi}{3} \Rightarrow \Theta = \frac{\pi}{6}$$

So:

$$A = 8 \left[ \frac{1}{3} \int_{0}^{\pi/6} \left( r_{\text{outer}}^2 - r_{\text{inner}}^2 \right) d\Theta \right]$$

$$= 4 \int_{0}^{\pi/6} \left( 16 \cos^2 2\Theta - 4 \right) d\Theta$$

$$= 4 \int_{0}^{\pi/6} \left( 8 + 8 \cos 4\Theta - 4 \right) d\Theta$$

$$= 4 \left[ 4\Theta + 2 \sin 4\Theta \right]_{0}^{\pi/6}$$

$$= 4 \left[ \left( 4 \cdot \frac{\pi}{6} + 2 \sin \frac{4\pi}{6} \right) - 0 \right]$$

$$= \frac{8\pi}{3} + 4\sqrt{3}$$

From symmetry, we know the total area is twice the shaded area.

There is an inner and outer curve.

$$A = 2 \left[ \frac{1}{2} \int_{\pi/2}^{\pi/12} \left( r_{\text{outer}}^2 - r_{\text{inner}}^2 \right) d\Theta \right]$$

$$A = \int_{\pi/12}^{\pi/2} \left( 2 \sin 2\Theta - 1 \right) d\Theta$$
\[ r = 1 + \sin \theta \]

\[ r = 1 - \cos \theta \]

- A curve in \( \mathbb{R}^I \) strikes only the red line
- A curve in \( \mathbb{R}^II \) strikes only the blue line

\[ \Rightarrow 2 \text{ integrals!} \]

Limits:

\[ \sin \theta = \cos \theta \]

\[ \sin \theta = \cos \theta \]

\[ \tan \theta = 1 \]

\[ \Rightarrow \theta = \frac{\pi}{4} \]

So:

\[ A_I = \frac{1}{2} \int_{0}^{\pi/4} r_I^2 \, d\theta \]

\[ A_{II} = \frac{1}{2} \int_{\pi/4}^{\pi/2} r_{II}^2 \, d\theta \]

\[ A_I = \frac{1}{2} \int_{0}^{\pi/4} (1 + \sin \theta)^2 \, d\theta \]

\[ A_{II} = \frac{1}{2} \int_{\pi/4}^{\pi/2} (1 + \cos \theta)^2 \, d\theta \]

and \[ A = A_I + A_{II} \]