Worksheet #9: Parametric Equations and Polar Coordinates.

I. Parametric Equations

1. Eliminate the parameter and find a Cartesian representation of the given curves. Then, make a sketch and indicate the positive orientation.
   
   a) \( x = 2t^2, \ y = 3t \)
   
   b) \( x = 4 \sin t + 1, \ y = 4 \cos t \)
   
   c) \( x = 4e^t, \ y = e^{3t} \)
   
   d) \( x = 2 \cos t, \ y = \sin t \)

2. Give a parameterization of the circle \( x^2 + (y-1)^2 = 4 \), that:
   
   a) Starts at \((0,3)\) when \(t=0\).
   
   b) Is traced out clockwise.
   
   c) Starts at \((2,1)\) and is traced out twice between \(t=0\) and \(t=1\).

3. Suppose \( x = at \) and \( y = bt \).
   
   a) Show that this curve is the line \( y = \frac{b}{a} x \).
   
   b) Find another parameterization of this curve that starts at \( y = b \) when \( t = 0 \).

4. Suppose \( x = t^3 - 3t, \ y = t^2 \).
   
   a) Find \( \frac{dy}{dx} \) in terms of \( t \).
   
   b) Find all horizontal and vertical tangent lines.
   
   c) Find the tangent line at \( t = 2 \).
   
   d) Find the tangent line at \((x,y) = (2,1)\).

5. Suppose \( x = t^2 - 6t + 1, \ y = \frac{1}{2}t^2 - t \)
   
   a) Find \( \frac{dx}{dt} \) in terms of \( t \).
   
   b) Find all vertical and horizontal tangent lines.
c) Find the tangent line at $t=1$.

d) Find the tangent line at $(x,y) = (1, 1\frac{\pi}{2})$.

II. Polar Coordinates

6. Express the point $(x, y) = (0, 1)$ in polar coordinates in 4 different ways.

7. Express the point $(x, y) = (-\sqrt{2}, -\sqrt{2})$ in polar coordinates in 4 different ways.

8. Express the point $(r, \theta) = (4, \frac{\pi}{3})$ in Cartesian coordinates.

9. Given the following curves $r = f(\theta)$, express the curves in Cartesian coordinates:

   a) $r = 4 \sec \theta$

   b) $r^2 = \tan \theta$

   c) $1 = \sin \theta \cos \theta$

   d) $r = 5$

10. Let $r = 2 \cos \theta$

   a) Find $\frac{dy}{dx}$ in terms of $\theta$.

   b) Find the vertical and horizontal tangent lines in Cartesian coordinates. Indicate the points of tangency in Cartesian Coordinates.

   c) Find the tangent line when $\theta = \frac{\pi}{6}$.

11. Repeat 10a), b) for $r = 2 + 2 \sin \theta$

12. Find the area inside the curve $r = \sqrt{\cos \theta}$ and inside the circle $r = \frac{1}{2}$.

13. Find the area of the region inside $r = 4 \cos 2\theta$ and outside $r = 2$.

14. Set up an integral that represents the area inside $r^2 = 2 \sin 2\theta$ and outside $r = 1$.

15. Set up an integral that represents the area between $r = 1 + \sin \theta$ and $r = 1 + \cos \theta$ in $\Omega$. 