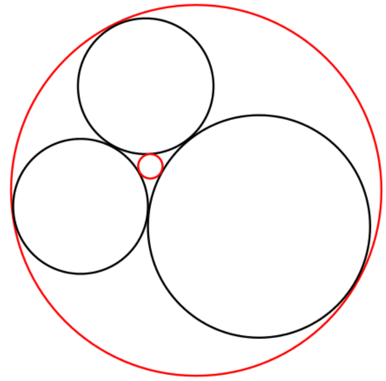
**What is… Kissing Circles?**
Presented by Carl Ahlborg on 7/9/15



Descartes' Theorem (Rene Descartes, 1643)
For every four kissing[1] circles, the radii of the circles satisfy

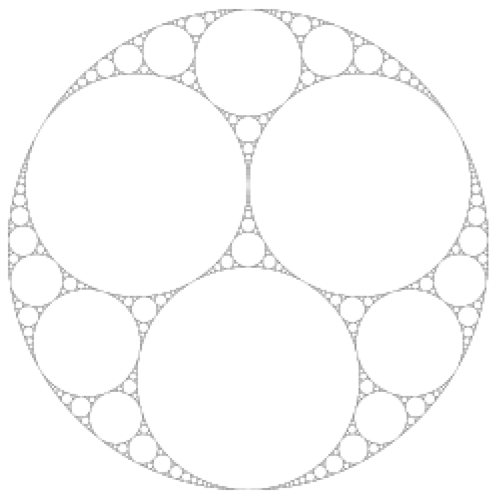
$$\left(\sum_{i=1}^{4} k_i\right)^2 = 2\sum_{i=1}^{4} {k_i}^2$$

where $k_i = \pm\frac{1}{r_i}$.

Therefore, given any three kissing circles, such as the black circles in the image to the right, Descartes' Theorem can be used to find the radius of a fourth kissing circle. However, there are two such circles tangent to the other three. It is noteworthy that $k$ is defined as positive or negative; a positive $k$ value corresponds to externally tangent circles while a negative $k$ value corresponds to internally tangent circles. Thus, the three black circles would be positive, the small red circle would be positive, and the large red circle would be negative.

Soddy
Frederick Soddy rediscovered this equation in 1936, and he published the theorem in the form of a poem titled *The Kiss Precise*. Soddy extended the theorem to spheres by incrementing $i$ to 5 instead of 4 in the summations.

Gosset
Thorold Gosset extended the theorem to arbitrary $n$ dimensions, once again resulting in an index of $n+2$ for the summations.



Appolonius
Apollonius of Perga considered around 200 BC the eight circles that are tangent to each of 3 given circles in general position. By continuing to apply Descartes theorem for a set of any three circles, one can create an *Apollonian packing*, pictured right.

Complex Descartes' Theorem (not attributed)
By expressing the coordinates of the center of a circle $(x,y)$ as the complex number $z = x + iy$, setting $k_i = \pm\frac{z}{r_i}$ again results in the same relationship as Descartes' Theorem.

References:
[1] "Descartes' Theorem." Wikipedia.
[2] J. Lagarias, C. Mallows, and A. Wilks, "Beyond Descartes Circle Theorem." 2001
[3] Achille Hui, "Proof of Descartes' Theorem." Math.stackexchange.com. (2014).

[1] Mutually tangent