THE OHIO STATE UNIVERSITY

Mathematics 5576H
Honors Number Theory
Autumn (odd numbered years)
5 credits

Catalog Description:
Elementary analytic and algebraic number theory, tracing its unifying role in the development of mathematics through history.

Prerequisite:
C or better in 4182H, or in both 2182H and 3345; or credit for 264H, or for both 263H and 345; or permission of department.

Purpose of Course:
The intention of this course is to present number theory, the "Queen of Mathematics" through its historical development. Being one of the oldest mathematical disciplines, number theory, in the course of its history, both benefited from and contributed to such major mathematical areas as geometry, algebra and analysis. These courses will be especially beneficial for honor students planning to pursue careers in mathematics, physics, computer science and education, but may be of interest to engineering students as well.

Text:
Vary, for example:

- *An Introduction to the Theory of Numbers*, 6th edition, by Hardy, Wright, Heath & Brown, published by Oxford, ISBN: 9780199219865.
- *An Introduction to the Theory of Numbers*, I. Niven, H.S. Zukkerman, H.L. Montgomery
- *Number Theory: An Introduction to Mathematics, Parts A and B*, by William A. Coppel, Springer-Velag.

Topics List:

1. Review of Egyptian and Mesopotamian Mathematics. Greek tradition. Three classical Greek problems (cube doubling, angle trisection, circle quadrature).
2. Famous irrationalities.
3. Continued fractions and applications thereof (quadratic surds, Pell's equation, Diophantine approximations, etc.)
4. More on diophantine approximation. Algebraic numbers. Liouville numbers. A glimpse into the Thue-Siegel-Roth Theorem.
5. Uniform distribution modulo one. Weyl criterion. Some important sequences. Pisot-Vijayaraghavan numbers. Formulation and discussion of Margulis' solution of Oppenheimer's conjecture.
6. Normal numbers. Champernoun's example. Almost every number is normal. Levy-Khinchine Theorem on normality of continued fractions.
7. Infinitude of primes. Euler's identity. Chebyshev's Theorem. Bertrand's Postulate. Dirichlet's Theorem on primes in progressions. Average rate of growth of classical number-theoretical functions.
8. Finite fields. Wedderburn's Theorem. Applications: Latin Squares and Cryptography.
9. Quadratic reciprocity.

10. Pythagorean triangles. Representation of integers as sums of squares. Quaternions, Cayley's octavas. Hurwitz' Theorem. Minkowski's geometry of numbers.
11. *p*-adic numbers, their construction and axiomatic characterization (Ostrowski's Theorem). Minkowski-Hasse principle.
12. Fermat's last theorem. Some easy cases. A glimpse into modern developments (elliptic curves, Mordell-Weil Theorem, etc.).