

---

---

# Algebraic Number Theory

---

---

## Instructor and Class Information

---

Lecturer: J. Cogdell

Course Number:

Office: MW 632

Lecture Room:

Phone: 2-8678

Lecture Times: 1:50

Email: cogdell.1

Office Hours:

## About Course Goals

---

### FORMAT

The course will meet three times a week for 55 minutes each meeting. Instructions will be mainly by lecture delivered by the instructor. It may also include occasional in-class discussion as well as short student presentations, particularly, by post-candidacy students.

### DESCRIPTION & GOALS

Math 7121 is a course in algebraic number theory. The main objects of study are number fields (finite extensions of the rational number field  $\mathbb{Q}$ ); their rings of integers, prime ideals, unique factorization into prime ideals, the ideal class group, and ramification; their invariants such as the different and discriminant, the class number and the regulator. We use a combination of local and global techniques. In particular we will study local fields, the real and complex fields, and the various  $p$ -adic fields that arise as completions of number fields, and their relation with algebraic number fields. The remaining time will be dedicated to a survey of class field theory. Class field theory centers on the following type of question: for a number field  $k$ , describe the finite abelian extensions  $K/k$ , and particularly the splitting of primes in these extensions, in terms of data internal to  $k$ .

### PREREQUISITES

Math 6112

## Textbook

---

### MAIN REFERENCE

Jim Milne: *Algebraic Number Theory*. Course notes from the University of Michigan. Available at <http://www.jmilne.org/math/CourseNotes/ant.html>.

### ADDITIONAL REFERENCES

E. Artin: *Theory of Algebraic Numbers*, in *Exposition by Emil Artin: A Selection*, Edited by: Michael Rosen, AMS, Providence, 2006.

S. Lang, *Algebraic Number Theory*, (1970). Springer GTM **110**.

J. Neukirch, *Algebraic Number Theory* (1992). Springer Grundlehren **322**.

## Assessments

---

### HOMEWORK ASSIGNMENTS

There will be periodic, probably weekly, problem sets, and a final problem set in lieu of a final exam. These will be handed out in class.

### CLASS PARTICIPATION AND ATTENDANCE

I expect attendance.

## Grading

---

### COURSE SCORE

A course score will be computed from the above assessments.

### LETTER GRADES

Letter grades will be determined based on the course score. The approximate minimum scores letter grades are 90% for an "A" or "A-", 80% for a "B+", 'B', or "B-", and 70% for a grade in the "C" range. The exact cut-off scores may vary depending on the difficulty of assignments.

## Weekly Schedule

---

Week 1	Integrality, rings of integers in a number field
Week 2	The trace form; discriminants of number fields
Week 3	Dedekind domains; localization
Week 4	Prime factorization of ideals; fractional ideals
Week 5	Factorizations in extensions
Week 6	The discriminant and different
Week 7	Finiteness of the class number
Week 8	Class number & Minkowski's geometry of numbers
Week 9	Dirichlet unit theorem; the regulator
Week 10	Cyclotomic fields
Week 11	Valuations; local fields
Week 12	Hensel's Lemma & Krasner's Lemma
Week 13	Ramified and unramified extensions of local fields
Week 14	Class field theory

## General Policies

---

### ACADEMIC MISCONDUCT

It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the

committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct ([http://studentaffairs.osu.edu/info\\_for\\_students/csc.asp](http://studentaffairs.osu.edu/info_for_students/csc.asp)).”

### **DISABILITY SERVICES**

Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; <http://www.ods.ohio-state.edu/>.