
Lie Groups and Their Representations

Instructor and Class Information

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| Lecturer: | Course Num.: |
| Office: | Lecture Room: |
| Phone: | Lecture Times: |
| Email: | Office Hours: |

About Course Goals

FORMAT

The course will meet three times a week for 55 minutes each meeting. Instructions will be mainly by lecture delivered by the instructor. It may also include occasional in-class discussion as well as short student presentations, particularly, by post-candidacy students.

DESCRIPTION & GOALS

This course provides students with a solid working knowledge in the theory of Lie groups and their representations. We will primarily focus on analytic and algebraic aspects of compact Lie groups through a wide variety of examples. These objects themselves and techniques used to study them are prevalent in several areas of mathematics. This course should be of interest to students working in or planning to work in representation theory, number theory or algebraic geometry.

PREREQUISITES

This section is open only to mathematics post-candidacy students and requires, in addition, the permission of the instructor.

Textbook

MAIN REFERENCE

Instructor Lecture Notes

W. Rossmann: *"Lie Groups: an introduction through linear groups"* Oxford Graduate Texts in Mathematics 2002. ISBN: 9780198596837.

ADDITIONAL REFERENCES

Lawrence Conlon: *"Differentiable Manifolds"*. Birkhäuser Boston, 2001. ISBN:0817641343.

A. Knapp: *"Lie Groups beyond an introduction"*. Birkhäuser Boston, 2002.

C. Chevalley: *"Theory of Lie Groups"* Princeton Landmarks in Mathematics ISBN: 9780691049908 *"Manifolds"*. Springer, 2002. ISBN:0387954481.

Assessments

CLASS PARTICIPATION, AND ATTENDANCE

Although attendance is not regularly monitored frequent absences are likely to be noted and may

factor into the grade in borderline cases discussions that arise from lecture material. Students are expected to attend all classes.

FINAL PRESENTATION

Students' grade will be solely based on a final in-class short presentation. A list of suggested topics will be published about three to four weeks before the end of classes and student presentations will be organized during the finals week.

Grading

COURSE GRADE

The grades for this course section are "satisfactory" (S) or "unsatisfactory" (U). A satisfactory outcome requires continued active participation in class (weighed about 20%) and is further based on the student's performance during the presentation (weighed about 80%).

Weekly Schedule

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|---------|---|
| Week 1 | Definitions and Examples: Lie Groups, Differential Forms, Vector Fields, Lie algebras |
| Week 2 | Correspondence between Lie groups and Lie algebras. Exponential map. Adjoint representation |
| Week 3 | Semisimple and Unipotent elements. Jordan decomposition. Solvable and nilpotent Lie groups. Lie's theorem. |
| Week 4 | Compact Lie groups. Maximal tori. Weyl group. |
| Week 5 | Compact Lie groups cntd. Root systems. |
| Week 6 | Classical (linear) Lie groups. Connectedness and simply-connectedness. |
| Week 7 | Classification of root systems. |
| Week 8 | Complex semisimple Lie groups and algebras. Cartan-Killing form. Compact real form of a complex Lie group. |
| Week 9 | Integration on compact Lie groups. Weyl's integration formula. |
| Week 10 | Representations. Complete reducibility theorem. Orthogonality of irreducible characters. |
| Week 11 | Peter-Weyl theorem. Classification of irreducible representations. |
| Week 12 | Weyl's character formula and its applications. |
| Week 13 | Homogeneous spaces. Construction of irreducible representations: Borel-Weil-Bott theorem. |
| Week 14 | Decomposition of tensor product of irreducible representations. Combinatorial formulae for tensor product multiplicities. |

General Policies

ACADEMIC MISCONDUCT

It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the

committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).”

DISABILITY SERVICES

Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; <http://www.ods.ohio-state.edu/>.