Gordon Prize Examination
February 24, 2001

1. Prove that there is a partition of \{1,2,\ldots,2001\} into 667 subsets such that each subset contains exactly 3 elements and, for each subset, the sum of its 3 members is 3003.

2. A *dolphin* is a chess piece that moves either one step forward, or one step to the right, or one step diagonally backwards to the left: Can a dolphin placed in the lower lefthand corner square of an 8 \times 8 chessboard run through all of the chessboard visiting each square exactly once?

3. Prove that there exists a circle in the Cartesian plane \(\mathbb{R}^2\) whose interior contains exactly 2001 lattice points (that is, points \((a,b)\in\mathbb{R}^2\) such that both \(a\) and \(b\) are integers).

4. Let \(P(x) = x^n + a_{n-1}x^{n-1} + \ldots + a_1x + 1\) be a polynomial such that \(a_{n-1},\ldots,a_1\) are nonnegative real numbers. Suppose that \(P(x)\) has \(n\) real roots. Prove that \(P(2) \geq 3^n\).

5. Define a *neighbor* of a square \(S\) on a standard 8 \times 8 chessboard to be any square that shares an edge with \(S\). In each square of the chessboard, a number is placed. Suppose that, for every square \(S\), the number placed in \(S\) is equal to the average of the numbers placed in the neighbors of \(S\). Prove that all of the numbers placed on the chessboard are equal.

6. Let \(a\), \(b\), and \(c\) be complex numbers such that at least two of them are distinct. Let \(z = \frac{-1}{2} + i\frac{\sqrt{3}}{2}\) and assume that \(a + bz + cz^2 = 0\). Show that \(a\), \(b\) and \(c\) are the vertices of an equilateral triangle in the complex plane.