Gordon Prize Exam

February 23, 2008

1. Do there exist 2008 distinct positive integers $n_1, n_2, \cdots, n_{2008}$ such that

$$2 = \frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_{2008}} \qquad ?$$

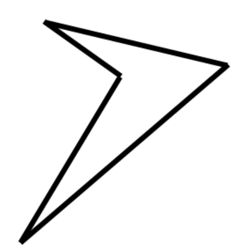
2. Chameleons on an island come in three colors. They wander and meet in pairs. When two chameleons of different colors meet, they both change to the third color. Given that the initial amounts of the chameleons of the three colors are 13, 15, and 17, show that it may not happen that, after a while, all of them acquire the same color.

3. Let $a_1, a_2, \cdots, a_6$ be positive real numbers with sum 1. Prove:

$$\left(\frac{1}{a_1} - 1\right)\left(\frac{1}{a_2} - 1\right)\cdots\left(\frac{1}{a_6} - 1\right) \geq 2008.$$

4. A **lattice point** in the Cartesian plane is a point with both coordinates integers. Three lattice points A, B, C are given. Must there exist three more lattice points A', B', C' such that the triangle $\triangle A'B'C'$ is similar to $\triangle ABC$ but with 5 times the area?

5. A **dart** is a non-convex 4-sided polygon in the plane. Can a convex polygon be partitioned into finitely many darts?



6. Let A and B be $n \times n$ real matrices. Suppose that $I - AB$ has an inverse, where I is the $n \times n$ identity matrix. Show that $I - BA$ also has an inverse.

Answer each problem on the separate sheet containing the statement of the problem, or on an additional sheet. "Answers" without appropriate reasoning are insufficient. **Put your secret code name at the top of each answer sheet.**

You may take this sheet with you when you leave. Be sure to hand in separately the cover sheet (with your name and secret code name).