2010 Gordon Prize examination

1. In the plane, consider an infinite strip of width d. (The region between two parallel lines.) Suppose every triangle of area 1 will fit inside the strip, after suitable translation and rotation. What is the minimum possible width d?

2. Let ABC be a triangle with acute angles α, β and γ such that

$$\tan(\alpha - \beta) + \tan(\beta - \gamma) + \tan(\gamma - \alpha) = 0.$$

Prove that ABC is isosceles.

3. The number 2010 is written as a sum of two or more positive integers. What is the maximum possible product of these integers?

4. Let A be a 2010×2010 matrix such that in every row and in every column, exactly two entries are equal to 1 and the rest are 0. Prove that the determinant of A is either 0 or $\pm 2^m$ where m is even.

5. Evaluate $\lim_{n\to\infty} n \sin(2\pi n! e)$.

6. Let α be a real number. Find $\lim_{n\to\infty} \begin{pmatrix} 1 & \alpha/n \\ -\alpha/n & 1 \end{pmatrix}^n$.