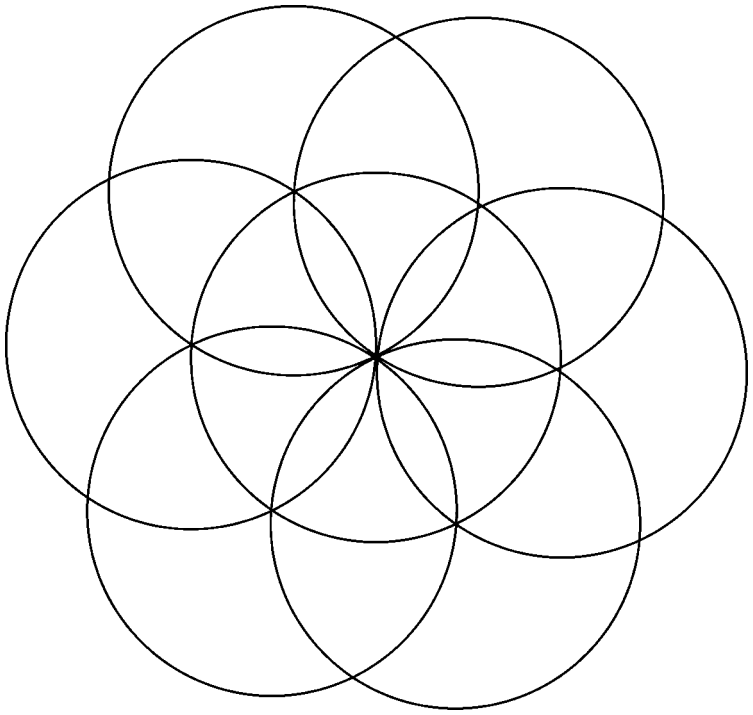
Rasor-Bereis Prize Examination

February 24, 1996

1. A **hexafoil** is constructed as follows: Draw a circle of radius 1. Draw six more circles of radius 1 whose centers are at the vertices of a regular hexagon inscribed in the first circle. The hexafoil consists of the six petals shaded in the picture. Compute the area of the hexafoil.



2. Prove that the number $24^{1996} - 14^{1996}$ is divisible by 19.

3. A positive integer is called a **zero-one-ite** if its decimal expansion consists only of 0's and 1's. Prove that every positive integer is a divisor of some zero-one-ite.

4. From a calculator, it seems that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$. Can you prove it rigorously?

5. Prove that $x^4 + 2x^2 + 2x + 2$ cannot be factored into two quadratic polynomials with integer coefficients.

6. A page (two sides) is torn from a paperback novel. The sum of the remaining page numbers is 15,000. Which page was torn out?

You may take this sheet with you.
Be sure to hand in separately the cover sheet
(with your name, rank, student number, and secret code name).
Put your secret code name at the top of each answer sheet.