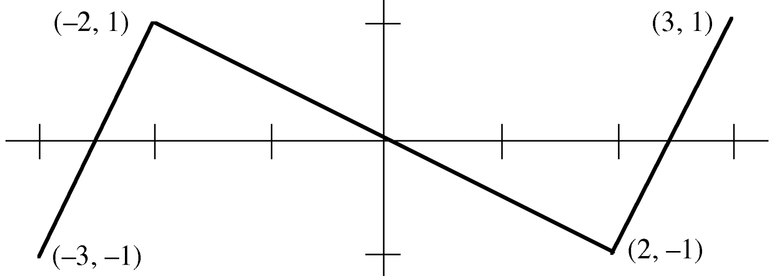
Rasor-Bareis Prize Examination

February 27, 1999

1. How many zeros are on the end of the decimal representation of $1^{1999}+2^{1999}+3^{1999}+4^{1999}$?

2. Suppose that the graph of $y = f(x)$ is defined on the closed interval $[-3, 3]$ and consists of three line segments as shown. Sketch the graph of the equation $f(x) = f(y)$, that is, the set of pairs (x, y) satisfying the equation. Label the coordinates of the corners.



3. Suppose the three angle bisectors in a triangle all have length > 1. Show that the triangle must have area $> 1/\sqrt{3}$.

4. Suppose the real number $(6 + \sqrt{37}\,)^{1999}$ is written out in decimal form. Show that at least the first thousand digits after the decimal point are zeros.

5. Does the equation $x^2 + y^2 - z^2 = 1999$ have infinitely many solutions in integers?

6. The *Fibonacci numbers* are defined by $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$, $n \geq 2$. Prove that any positive integer may be written as a sum of (one or more) *different* Fibonacci numbers. (Note that F_1 and F_2 are *not* considered different.)

You may take this sheet with you.
Be sure to hand in separately the cover sheet
(with your name, rank, student number, and secret code name).
Put your secret code name at the top of each answer sheet.