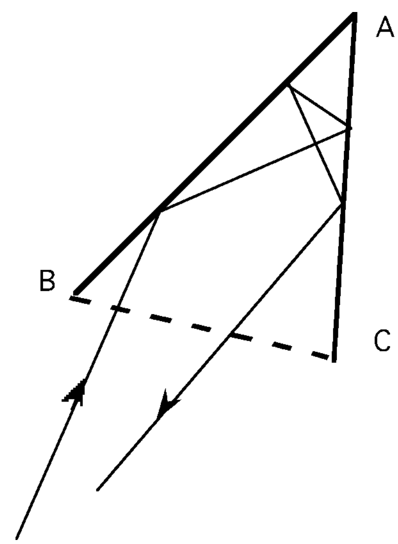
Rasor-Bareis Prize Examination

February 12, 2000

1. Let $P_1 P_2 P_3 \cdots P_{12}$ be a regular dodecagon. Prove or disprove that the diagonals P_1P_9, P_2P_{11}, and P_4P_{12} have a point in common.

2. An angle is formed by two mirrors. If a ray of light enters the angle, and does not hit a corner, then must it leave the angle? More precisely: Suppose the ray crosses side BC into the triangle, and reflects with angle of incidence equal to angle of reflection whenever it hits sides AB and AC. Assume that the ray does not exactly hit any of the points A, B or C. Then must it eventually cross side BC back to the outside?



3. Suppose the plane is colored red and blue. (That is, there are two disjoint sets R, B whose union is the plane. Elements of R are said to be "colored red" and elements of B "colored blue.") Prove that there is some rectangle in the plane with all four vertices of the same color.

4. Is there a polynomial p with integral coefficients satisfying

(a) $p(1) = 1$, $p(2) = 2$, $p(3) = 123$?

(b) $p(1) = 1$, $p(2) = 2$, $p(3) = 124$?

5. For which positive integer n does the quotient

$$\frac{\log 2\ \log 3\ \log 4\ \cdots\ \log n}{10^n}$$

take its minimal value? ["log" denotes the logarithm base 10.]

6. Three runners run along three parallel racetracks in the plane. Each runner runs with constant speed, but the runners may have different speeds from each other. At a certain moment, the triangle formed by the three runners had area 2; five seconds later the area became 3; what will this area be in five more seconds?

You may take this sheet with you.
Be sure to hand in separately the cover sheet
(with your name, rank, student number, and secret code name).
Put your secret code name at the top of each answer sheet.