1. Solve: \( \sin^{2008} x + \cos^{2008} x = 1. \)

2. Let three adjacent squares be given, as in the diagram.
   Show that \( \angle ACB + \angle AEB + \angle AGB = 90^\circ. \)

3. Note that 2 can be written as a sum of the reciprocals of four distinct positive integers:
   \[ 2 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6}. \]
   Can 2 be written as a sum of the reciprocals of 2008 distinct positive integers:
   \[ 2 = \frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_{2008}} \]

4. Find all rational functions \( f(x) \) such that \( f(x^2 - x) = f(x^2 + x) \) for all real \( x. \)

5. Let \( x_1, x_2, \ldots, x_n \) be distinct integers \( > 1. \) Prove:
   \[ \left( 1 - \frac{1}{x_1^2} \right) \left( 1 - \frac{1}{x_2^2} \right) \cdots \left( 1 - \frac{1}{x_n^2} \right) > \frac{1}{2}. \]

6. Suppose \( x_1 > x_2 > \ldots \) is a decreasing sequence of real numbers. Suppose
   \[ x_1 + \frac{x_4}{2} + \frac{x_9}{3} + \cdots + \frac{x_n^2}{n} < 1 \]
   for all \( n. \) Show that
   \[ x_1 + \frac{x_2}{2} + \frac{x_3}{3} + \cdots + \frac{x_n}{n} < 3. \]

Answer each problem on the separate sheet containing the statement of the problem, or on an additional sheet. “Answers” without appropriate reasoning are insufficient.

**Put your secret code name at the top of each answer sheet.**

You may take this sheet with you when you leave. Be sure to hand in separately the cover sheet (with your name and secret code name).