Rasor-Bareis Prize Examination

February 21, 2009

1. Find all prime numbers among 101, 10101, 1010101, 101010101, ...

2. Is there a differentiable function $f(x)$ defined for $x > 0$, satisfying $f'(x) = f(x+1)$ for all $x > 0$, and such that $\lim_{x\to\infty} f(x) = \infty$?

3. A regular 2009-gon P is triangulated by diagonals, which means that several line segments are drawn, each joining two of the vertices of P, so that no two segments intersect except as at the end points, and that all the regions formed are triangles. Show that among the triangles thus obtained there is exactly one acute triangle.

4. Suppose $n \geq 3$ lines are drawn in the plane in general position; that is, no two of the lines are parallel, and no three of them cross at a point. Show that among the regions formed, at least one is a triangle.

5. Find $\sin 2\theta$ if $\sin^6\theta + \cos^6\theta = 2/3$.

6. Assume that your calculator is broken so that you can only add and subtract real numbers and compute their reciprocals. How can you use it to compute products?