

## 2010 Rasor-Bareis Prize examination

1. In the plane, consider an infinite strip of width  $d$ . (The region between two parallel lines.) Suppose every triangle of area 1 will fit inside the strip, after suitable translation and rotation. What is the minimum possible width  $d$ ?
2. Given 2010 points in the plane, does there exist a straight line having 1005 points on each side of the line?
3. The number 2010 is written as a sum of two or more positive integers. What is the maximum possible product of these integers?
4. Solve in integers the equation  $x^2 + y^2 + z^2 = 2xyz$ .
5. Each vertex of a regular 2010-gon is assigned a positive real number. Suppose each of these numbers is either the arithmetic mean or the geometric mean of its two neighbors. Prove that all these numbers are equal to each other.
6. Consider an increasing sequence of positive integers

$$1 = a_1 < a_2 < a_3 < a_4 < \dots$$

with  $a_1 = 1$  and  $a_n \leq 2a_{n-1}$  for all  $n \geq 2$ . Prove that any positive integer not belonging to this sequence is representable as a sum of two or more distinct elements from the sequence.