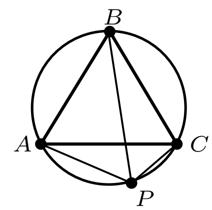
2011 Rasor-Bareis Prize examination

1. The vertices of a regular 2011-gon are colored in two colors. Prove that some three vertices of the same color form an isosceles triangle. (A triangle is called *isosceles* if at least two of its sides have equal length.)

2. Let $g(x) = a_0 + a_1x^{r_1} + a_2x^{r_2} + \ldots + a_nx^{r_n}$, $x > 0$, where $n \in \mathbb{N}$, $a_0, a_1, \ldots, a_n$ are nonzero real numbers and $r_1, \ldots, r_n$ are distinct nonzero real numbers. Prove that g has at most n zeroes in $(0, \infty)$.

3. Let ABC be an equilateral triangle and P be a point on the arc AC of the circumscribed circle. Prove that $|BP| = |AP| + |CP|$.



4. Prove that $a^4 + b^4 + c^4 \geq abc(a + b + c)$ for all $a, b, c \in \mathbb{R}$.

5. Solve the equation $$\cfrac{x}{2 + \cfrac{x}{2 + \ddots\, 2 + \cfrac{x}{2 + \cfrac{x}{1+\sqrt{1+x}}}}} = 1,$$ where "2" appears 2011 times.

6. Let a polynomial $p(x, y)$ satisfy $p(x + y, y - x) = p(x, y)$ for all $x, y \in \mathbb{R}$. Prove that p is constant.