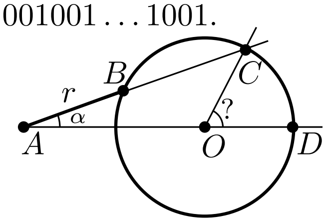
**2014 Rasor-Bareis examination problems**

**1.** Prove that there does not exist a prime integer of the form $1001001\ldots1001$.

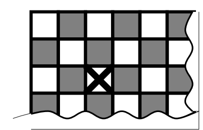
**2.** Suppose $|AB| = r$, the radius of the circle. If $\angle OAB = \alpha$, find $\angle DOC$.

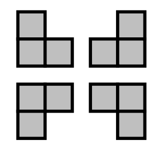



**3.** Let $a$, $b$, and $c$ be the lengths of the three sides of a triangle. Prove that

$$\frac{a}{b+c-a} + \frac{b}{a+c-b} + \frac{c}{a+b-c} \geq 3.$$

**4.** A cell is removed from a $2^n \times 2^n$ chessboard. Prove that the remaining part of the board can be tiled (that is, covered without overlapping) by $L$-shapes, as in the picture.


The board


The $L$-shapes

**5.** For any seven real numbers $y_1, \ldots, y_7$ prove that there are two of them, $y_i$ and $y_j$ with $i \neq j$, satisfying $\left|\frac{y_i - y_j}{1 + y_i y_j}\right| < \frac{1}{\sqrt{3}}$.

**6.** A rectangular pool table $ACDF$ has size $2024 \times 1111$. This table has six pockets located at the corners and at the midpoints of the longer sides, as indicated by $A, B, C, D, E, F$ in the figure. A ball is hit from corner $A$ at a $45°$ angle, and bounces off the edges until it falls into one of the pockets. Which pocket will it end in? Explain how you arrived at your answer. (Each pocket has diameter $d < 1$ and the ball has diameter $< d$; the ball moves at constant speed, making perfect bounces without friction or spin, until it hits a pocket.)