2018 Rasor-Bareis examination problems

1. Prove that the decimal integer of the form $20182018\ldots 201820182019$ cannot be a perfect square.

2. Prove that $\displaystyle\sum_{1\le m<n\le 2018} \frac{1}{mn}$ is not an integer.

3. Evaluate $\displaystyle\int_0^\pi \operatorname{arccot}(\cos x)\,dx$.
(Here "arccot" stands for arccotangent, the inverse of the cotangent function.)

4. Prove that the perpendicular bisector of the line joining the feet of two altitudes of a triangle bisects the third side of the triangle.

PICTURE

5. Let $0 < \alpha, \beta < \pi/2$ and assume that $\sin^2\alpha + \sin^2\beta = \sin(\alpha+\beta)$. Prove that $\alpha+\beta = \pi/2$.

6. Prove that for any positive integer n, $(2n+1)^n \ge (2n)^n + (2n-1)^n$.