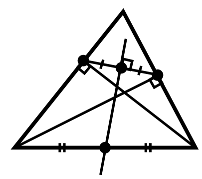
2018 Rasor-Bareis examination problems

1. Prove that the decimal integer of the form $20182018\ldots201820182019$ cannot be a perfect square.

2. Prove that $\displaystyle\sum_{1\le m<n\le 2018}\frac{1}{mn}$ is not an integer.

3. Evaluate $\displaystyle\int_0^{\pi}\operatorname{arccot}(\cos x)\,dx$.
(Here "arccot" stands for arccotangent, the inverse of the cotangent function.)

4. Prove that the perpendicular bisector of the line joining the feet of two altitudes of a triangle bisects the third side of the triangle.



5. Let $0<\alpha,\beta<\pi/2$ and assume that $\sin^2\alpha+\sin^2\beta=\sin(\alpha+\beta)$. Prove that $\alpha+\beta=\pi/2$.

6. Prove that for any positive integer n, $(2n+1)^n\ge(2n)^n+(2n-1)^n$.