Fresnel Integrals

Definition:

$$C(x) = \int_0^x \cos(t^2)\,dt \qquad S(x) = \int_0^x \sin(t^2)\,dt$$

Applications to Optics

The Wave Equation: $\nabla^2 F = \frac{1}{c^2}\frac{\partial F}{\partial t}$

Consider solutions of the form: $F = \Psi(\vec{x})e^{-i\omega t}$ where Ψ satisfies the Helmholtz equation $\nabla^2\Psi + k^2\Psi = 0 \quad k = \frac{\omega}{c}$

Kirchhoff Integral Theorem: If Ψ satisfies the Helmholtz equation then

$$\Psi(\vec{p}) = \frac{-1}{4\pi}\int \Psi\nabla\left(\frac{e^{ikr}}{r}\right) - \frac{e^{ikr}}{r}\nabla\Psi \;\; dA \quad where\ r = |\vec{x} - \vec{p}|$$

Make the approximations:

$$\nabla\left(\frac{e^{ikr}}{r}\right) \approx ik\,\frac{e^{ikr}}{r}\,\hat{r} \qquad \nabla\Psi \approx -ik\Psi\hat{r}$$

Huygens-Fresnel Equation

$$\Psi(\vec{p}) = \frac{k}{2\pi i}\int \Psi(\vec{x})\frac{e^{ikr}}{r}\;\; dA$$

Euler Spiral

$\vec{r}(t) = \langle C(t), S(t)\rangle$