

WHAT ARE KLOOSTERMAN SUMS?

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Abstract

In his study of quaternary quadratic forms, Kloosterman refined the Hardy-Littlewood method and introduced the sums which now bear his name. The (classical) Kloosterman sums are of the form

$$S(m, n; c) = \sum_{\substack{x(c) \\ (x, c) = 1}} e\left(\frac{mx + n\bar{x}}{c}\right),$$

where $c > 0$, m, n are integers, $x\bar{x} \equiv 1 \pmod{c}$ and $e(z) = \exp(2\pi iz)$. We will discuss algebraic aspects of Kloosterman sums, such as the multiplicativity property, Kloosterman's bound and Weil's bound. We will also discuss the connection of Kloosterman sums with automorphic forms, such as the spectral theory of Kloosterman sums (the Petersson/Kuznetsov formula) and its application on bounds of Fourier coefficients of cusp forms.