What is integration in terms of elementary functions?

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Abstract

Elementary functions are the ones studied in freshman calculus: the polynomial, trigonometric, exponential, and logarithmic functions, and combinations of these. A classical theorem of Liouville shows that integrals like $\int e^{-x^2} dx$ are not elementary. These ideas have been developed into algorithms that take an elementary function as input, and either provide a closed form for the antiderivative, or prove that the antiderivative is not elementary. These ideas led to the theory of differential fields. The proofs that antiderivatives of some elementary functions are not elementary are parallel to proofs that some polynomials are not solvable by radicals, as in Galois theory.