A SURVEY OF IRREDUCIBLE MORPHISM AND ITS DEGREES.

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Let $A$ be an artin algebra, denote by $\text{mod} \ A$ the category of finitely generated left $A$–modules, and by $\text{ind} \ A$ the full subcategory of $\text{mod} \ A$ consisting of one representative of each isomorphism class of indecomposable $A$–modules.

Since the notion of an irreducible morphism was introduce by M. Auslander and I. Reiten, it has played an important role in representation theory of artin algebras.

On the other hand, it is well-known that if $M$ is an indecomposable $A$–module then $\text{End}(M)$ is a local ring and the radical of $\text{End}(M)$ consists of all the non-isomorphisms. This definition can be extended to any module $M, N \in \text{mod} \ A$ to define the radical of $\text{Hom}_A(M, N)$. This radical, denote by $R(X, Y)$, is called the Jacobson radical of $\text{mod} \ A$, also been referred as $R(\text{mod} \ A)$.

In 1992, S. Liu introduced in [Liu92] the notion of degree of an irreducible morphism in the module category of an artin algebra inspired by the problem of when the non-zero composition of $n$ irreducible morphisms between indecomposable $A$–modules belongs to $R^{n+1}(\text{mod} \ A)$. Since then, many results has arosed in the theory of degrees of irreducible morphisms. For instance, the advances made by C. Chaio and her collaborators in recent years [Cha14].

In this session, we will talk about irreducible morphisms, the notion of degree of an irreducible morphism, and we will mention some problems in representation theory of artin algebras which the theory of degrees has helped to solve. Also, we will mention how the study of irreducible morphisms and radicals, has been amply treatment in many contexts such that for Auslander- Reithen theory relative to functorially resolving subcategories, following the work made by Shipping Liu.

REFERENCES
