While the notions of Baer, quasi-Baer, and extending modules are related to each other, it is well known that these are distinct notions in general. We discuss a number of instances under which these notions coincide with each other and other instances when these are distinct and disparate from each other. As a consequence, for a certain module $M$ over a commutative ring and a fixed injective hull $E(M)$, we study the coincidence, similarities and contrasts between Baer module hull, the quasi-Baer module hull, and the extending module hull of $M$.

We show that for an essential extension $V_R$ of a nonsingular cyclic module over a commutative ring $R$, the notions of $V_R$ being Baer, quasi-Baer, quasi-continuous, extending, and FI-extending, or strongly FI-extending, coincide with each other. Using this result, we prove the existence and coincidence of the Baer, the quasi-Baer, the extending hulls of any nonsingular cyclic module over a commutative ring. As application, such module hulls are explicitly described. As a byproduct, when $R$ is a commutative semiprime ring, all intermediate Baer (extending, quasi-continuous) ring between $R$ and $Q(R)$, where $Q(R)$ is the maximal (right) ring of quotients of $R$.

A characterization of a commutative domain to be Prüfer is obtained by using extending module hulls of certain modules. A number of examples which exhibit the disparities and differences between Baer hulls, quasi-Baer hulls, extending hulls, and FI-extending hulls of a module are provided.

(This is a joint work with S. Tariq Rizvi)