On the role of integration by parts in the analysis of the Cauchy-Leray integral

Loredana Lanzani
Syracuse University

Abstract. A few years ago, the speaker and E. M. Stein [LS-3] proved boundary $L^p$-boundary regularity, for any $1 < p < \infty$, of the Cauchy-Leray transform for strongly $\mathbb{C}$-linearly convex domains whose boundary is “near” $C^2$-smooth (specifically, in the class $C^{1,1}$). Recently the same authors proved [LS-1] that both assumptions ($C^{1,1}$-regularity and strong $\mathbb{C}$-linear convexity) are optimal by identifying two simple domains, each missing one of the two aforementioned assumptions, for which $L^p$-regularity of the Cauchy-Leray transform fails for any $1 \leq p \leq \infty$. In this talk we will report on the just-finished paper [LS-2], in which we show how the unbounded operators of [LS-1] rise as boundary values of the Cauchy-Leray integral. For sufficiently “nice” domains (such as those treated in [LS-1]), this convergence up to the boundary follows from a general paradigm, namely the fact that the Cauchy-Leray integral kernel is “essentially” a derivative: this leads to a global and intrinsic integration by parts, which in turn shows that, on a dense subset of $L^p(bD)$ the Cauchy-Leray integral can be represented by means of less singular operators (whose integral kernels are in fact absolutely convergent), for which the claimed boundary convergence can then be easily proved. As we shall see, the poor regularity (or poor convexity) of the two examples that will be discussed in this talk, limit the applicability of this general paradigm and a different approach will be required.

REFERENCES


