

Title: Ergodic Actions and Characters of Discrete Groups

Abstract. In 1980s Vershik noticed that for almost each extreme character χ on S_∞ , the group of permutations of \mathbb{N} with finite supports, there exists an ergodic S_∞ -invariant measure μ on the Baire space $\mathbb{N}^{\mathbb{N}}$ such that $\chi(g) = \mu(\text{Fix}(g))$ for every $g \in S_\infty$, where $\text{Fix}(g) = \{x : g(x) = x\}$. Vershik also *conjectured* that characters of any reasonably rich group G must come in the form $\chi(g) = \mu(\text{Fix}(g))$ for some ergodic action.

We will start the talk by discussing Vershik's results on characters of S_∞ and by proving that the function $\chi(g) = \mu(\text{Fix}(g))$ always defines a character. We will then show that Vershik's conjecture holds for two classes of groups: Thompson's groups $F_{n,1}$ and full groups of hyperfinite equivalence relations arising from minimal \mathbb{Z} -actions on the Cantor set. For Thompson's groups, we will use the fact that the action of $F_{n,1}$ on $X = (0, 1)$ is compressible and, as a result, admits no invariant measure to show that these groups have no non-trivial characters. The lack of characters has implications for the structure of invariant random subgroups. This talk is based on joint work with Artem Dudko.