Boundary regularity for the nonlinear p-parabolic and porous medium equations

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Abstract. Consider a bounded domain $\Omega \subset \mathbf{R}^n$ and $f \in C(\partial\Omega)$. In the Dirichlet problem we want to find a harmonic function u in Ω with takes the boundary values f, i.e. $u \in C(\overline{\Omega})$ and $u = f$ on $\partial\Omega$.

Without further regularity on Ω this problem typically lacks a solution. Perron introduced so-called Perron solutions $u = Pf$ to solve the Dirichlet problem in a weaker generalized sense. A natural question is then which boundary points are regular: x_0 is *regular* if

$$\lim_{\Omega \ni y \to x_0} Pf(y) = f(x_0) \quad \text{for all } f \in C(\partial\Omega).$$

A *barrier* at x_0 is a superharmonic function u (i.e. $-u$ is subharmonic) such that

$$\lim_{\Omega \ni y \to x_0} u(y) = 0 \quad \text{and} \quad \liminf_{\Omega \ni y \to x} u(y) > 0 \text{ for every } x \in \partial\Omega \setminus \{x_0\}.$$

Lebesgue (1924) showed that a boundary point is regular if and only if it has a barrier. This characterization has been extended to (the nonlinear elliptic) p-harmonic functions on metric spaces (Björn–Björn, 2006). The corresponding characterization for the heat equation is due to Bauer (1962).

In this talk the main focus is on the following three nonlinear parabolic (a.k.a. evolution) equations

$$\begin{aligned}
u_t &= \Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u), && \textit{the } p\textit{-parabolic equation}, \ p > 1,\\
u_t &= |\nabla u|^{2-p}\Delta_p u, && \textit{the normalized } p\textit{-parabolic equation}, \ p > 1,\\
u_t &= \Delta(u^m), && \textit{the porous medium equation}, \ m > 1.
\end{aligned}$$

I will discuss boundary regularity and in particular how regularity can be characterized using barriers for these three equations.

This talk is based on several joint papers with Jana Björn, Ugo Gianazza, Mikko Parviainen and Juhana Siljander.