Richard Durrett, Duke U.

Exact solution for a metapopulation version of Schelling's model

Abstract. In 1971, Schelling introduced a model in which families move if they have too many neighbors of the opposite type. In this paper we will consider a metapopulation version of the model in which a city is divided into N neighborhoods each of which has L houses. There are ρNL red families and ρNL blue families for some $\rho < 1/2$. Families are happy if there are $\leq \rho_c L$ families of the opposite type in their neighborhood, and unhappy otherwise. Each family moves to each vacant house at rates that depend on their happiness at their current location and that of their destination. Our main result is that if neighborhoods are large then there are critical values $\rho_b < \rho_d < \rho_c$, so that for $\rho < \rho_b$ the two types are distributed randomly in equilibrium. When $\rho > \rho_b$ a new segregated equilibrium appears; for $\rho_b < \rho < \rho_d$ there is bistability, but when ρ increases past ρ_d the random state is no longer stable. When ρ_c is small enough, the random state will again be the stationary distribution when ρ is close to $1/2$. If so, this is preceded by a region of bistability.